AN INVESTIGATION OF AN AUTOMATIC
THROTTLE DEVICE DURING TURNING FLIGHT

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AN INVESTIGATION OF AN AUTOMATIC THROTTLE

DEVICE DURING TURNING FLIGHT

by

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ABSTRACT

This thesis is a continuation of a series of studies on automatic throttle compensation systems begun in 1963 by Lieutenant Commander G. R. Bell. [1] Previous investigations have been concerned with flight in a wings level attitude. This study will consider the effects of turning flight on the system. The system under consideration here is the Specialties, Incorporated, Automatic Power Compensator, (APC), an automatic throttle in use in the F-8 and F-4 aircraft. It utilizes feedbacks of angle of attack and normal acceleration. It was found important to differentiate between spurious perturbations and those perturbations from steady state introduced due to the aircraft entering a turn.

This study was conducted by Lieutenant Richard D. Hartman at the United States Naval Postgraduate School, Monterey, California. The assistance given by Professor E. J. Andrews is greatly appreciated by the writer.

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LIST OF ILLUSTRATIONS

TABLE OF SYMBOLS

B	Moment of inertia about the Y-axis
c	Mean geometric chord
C_D	Drag coefficient
C_{D_α}	Change in drag coefficient with change in angle of attack
C_L	Lift coefficient
C_{L_α}	Change in lift coefficient with change in angle of attack
C_m	Pitching moment coefficient
C_{m_α}	Change in pitching moment coefficient with change in angle of attack
C_{m_η}	Change in pitching moment coefficient with change in elevator deflection
g	Acceleration due to gravity
m	Aircraft mass
n	Increment of normal acceleration
S	Wing area
ΔT	Change in thrust
U, V	Aircraft velocity
u	Velocity perturbation in x-direction
w	Velocity perturbation in z-direction
W	Aircraft gross weight
α	Change in body angle of attack
Θ	Change in pitch attitude
ϕ	Angle of bank
η	Change in elevator deflection

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Subscripts

a   Denotes conditions with wings level

o   Denotes initial trim conditions

t   Denotes conditions during turning flight

c   Denotes command quantity

r   Denotes reference quantity

1.   Introduction.

This study is one of a series being conducted at the United States Naval Postgraduate School to evaluate the use of automatic throttle compensation systems in modern high performance jet aircraft. The need for automatic throttle compensation was discussed thoroughly in the previous investigations. [1, 2] Briefly, the slope of the thrust required versus airspeed curve is negative at the approach airspeed for modern jet aircraft. This means that a loss of airspeed may rapidly develop into an uncontrolled sink rate. Aircraft with this characteristic are said to have negative static airspeed stability. Positive airspeed stability must be provided by proper use of the throttle. An automatic throttle compensation system relieves the pilot of the task of providing this airspeed stability. With an automatic throttle system keeping a constant airspeed, the pilot can concentrate on maintaining his altitude or glide slope by use of the elevator.

There are various combinations of errors in airspeed, pitch attitude, angle of attack, and normal acceleration that can be used as inputs to an automatic throttle device. Direct use of these feedbacks and their derivatives and integrals provide the final throttle command to the engine for the desired change in thrust. The system under consideration here utilizes feedback of angle of attack and normal acceleration. A block diagram of this system is shown in Figure 1. The transfer functions, also given in Figure 1, are those from Reference 3.

2.   Discussion.

Previous studies conducted at the U. S. Naval Postgraduate School

considered flight in a wings level attitude. It is the intent of this study to extend the analysis to include turning flight and to investigate the possible additional inputs to the basic automatic throttle system that may be required. The aircraft parameters used are for the F-8, an operational U.S. Navy swept-wing fighter. Previously, the equations involved were simulated on an analog computer and solutions obtained in this manner. For this investigation, use was made of the 1604 Control Data Corporation digital computer at the U. S. Naval Postgraduate School. A program titled "Runge-Kutta Solution of Ordinary Differential Equations with Built-in Input and Output Routines" was used to provide solutions for the equations.

Since the digital computer had not been used in References 1 and 2, the response of the basic airframe in a wings level attitude was first examined. This preliminary investigation was conducted with and without the automatic throttle compensator in use. When the results of this investigation showed close agreement with other results, turning flight was investigated.

This analysis is confined to the longitudinal mode of operation and the linearized three-degree-of-freedom, body axes, equations of motion derived from small perturbation theory are utilized. These equations as derived in Reference 4 are:

X-Force

$$\dot{u} = \frac{X_u}{m} u + \frac{X_w U_o}{m} \propto - g \theta + \frac{X_{\Delta T}}{m} \Delta T$$

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Z-Force

$$\dot{\alpha} = \frac{Z_u}{mU_o} u + \frac{Z_w}{m} \alpha + \dot{\theta} + \frac{Z_\eta}{mU_o} \eta$$

Moment

$$\ddot{\theta} = \frac{M_u}{B} u + \frac{M_{\dot{w}}U_o}{B} \dot{\alpha} + \frac{M_w U_o}{B} \alpha + \frac{M_\theta}{B} \dot{\theta} + \frac{M_\eta}{B} \eta + \frac{M_{\Delta T}}{B} \Delta T$$

The values for the stability derivatives in the above equations were obtained from Reference 3 and are given in Table I. These values are for the F-8 aircraft in landing configuration at approach speed. Several of these parameters are functions of C_L and C_D and would change values if found advisable to change the aircraft angle of attack while in a turn. Utilizing the values of the parameters and their basic definitions, they are developed in Appendix I as functions of C_L and C_D.

Curves of C_L and C_D versus angle of attack in Reference 5 were used to obtain the plot of C_L versus C_D shown in Figure 2. An equation of this plot is derived in Appendix II and found to be

$$C_D = .955 - 2.12(C_L) + 1.44(C_L)^2$$

In order to write a program for the digital computer a flow graph of the system to be analyzed is first developed. Such a flow graph for the airframe and automatic throttle combined is shown in Figure 3. The program for the digital computer developed from this flow graph is shown in Figure 4. Minor variations in this program and the use of initial conditions allows for an analysis of the system under a great variety of conditions.

3. Flight with Wings Level.

Aircraft Without Automatic Throttle

By eliminating the change of thrust inputs to the moment and X-force equations, the automatic throttle will be disconnected from the aircraft equations of motion. This was easily effected by changing several cards of the program of Figure 4. The response of the airframe to inputs of an initial velocity gust, an initial angle of attack and a step input of elevator was analyzed.

Figures 5a, 5b, and 5c show the response of the airframe velocity, angle of attack and pitch attitude respectively of an initial condition of velocity equal to ten feet per second. From this response, the phugoid period is found to be 33 seconds. This is in close agreement with that obtained in Reference 1 which was 33.5 seconds.

The airframe velocity, angle of attack and pitch attitude response to an initial angle of attack of two degrees is shown in Figures 6a, 6b, and 6c. The short period mode measured from Figure 6b is 6 seconds. This value also agrees with that found in Reference 1 which was 6.06 seconds.

Figures 7a, 7b, 7c, and 7d show the airframe response to a step input of one degree of elevator. A relationship between the angle of attack and elevator can be obtained from Figure 7b. A similar relationship, developed in Appendix III, agrees with that from Figure 7b. This relationship is:

$$\eta = .46 \, \alpha$$

Aircraft With Automatic Throttle

The response of the aircraft with the automatic throttle connected was investigated with the disturbance of an initial velocity gust of ten feet per

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second. This disturbance was introduced with no elevator in the system and with the elevator function developed above as a forcing function in the system. The aircraft velocity and angle of attack and the engine change of thrust responses with no elevator introduced are shown in Figures 8a, 8b, and 8c. These same responses with the elevator input is shown in Figures 9a, 9b, and 9c. The use of the input of elevator as a function of angle of attack reduced the required change of thrust by a small amount. Use of this elevator was made during the rest of the analysis.

The velocity, angle of attack and change of thrust responses to an initial condition of two degrees to the aircraft pitch attitude is shown in Figures 10a, 10b, and 10c.

The aircraft response with and without the automatic throttle as shown in Figures 5 through 10 agrees with the results obtained in References 1, 2, and 3. This indicated that the use of the digital program simulates the system to be analyzed as well as the analog computer utilized in the previous investigations.

4.   Turning Flight at Constant Altitude.

As an aircraft enters a turn and maintains an angle of bank, more lift must be generated in order to maintain altitude. This is due to the fact that the aircraft lift is no longer acting vertically, but at an angle ϕ, while the weight, of course, still acts vertically. The increased lift required can be generated by increasing the velocity while maintaining the same angle of attack or by increasing the angle of attack while maintaining the same velocity or by a combination of an increase in velocity and angle of attack.

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Figure 11 is a plot of the thrust required for flight for angles of bank up to thirty degrees. The calculations for this plot are shown in Appendix IV.

A cross-plot of the change in thrust against angle of bank at constant angle of attack is plotted on Figure 12. An equation of change in thrust as a function of angle of bank was developed from this curve and found to be

$$\Delta T = .05 \phi + .855 \phi^2$$

As an aircraft maintains altitude in a turn by generating more lift, the normal acceleration must necessarily increase. This increase in normal acceleration, n, as a function of angle of bank is easily found as follows:

$$\cos \phi = \frac{1}{1 + n}$$

$$1 + n = \frac{1}{\cos \phi}$$

$$n = \sec \phi - 1$$

In order to keep the automatic throttle from decreasing the thrust because of this increase in normal acceleration, the base for the normal acceleration loop must be changed. This can be accomplished by putting another input into this loop or "biasing" the loop. Figure 13 shows the block diagram of the system whith the expected input changes as the aircraft goes into a turn. The angle of attack loop can also be biased if it is decided to change the angle of attack in a turn.

To investigate the use of an automatic throttle device in turning flight, the perturbations introduced to the three longitudinal equations of motion due to the turn must be determined. An analysis of the possible velocity and angle of attack perturbations is made in Appendix V. After an aircraft enters

a turn, if the proper perturbations are introduced and the throttle and elevator are coordinated in such a way as to correct for these disturbances, a steady state should be reached at which the angle of attack, velocity and thrust level out at the new desired values.

It was decided to make the turn at a constant angle of attack, thereby eliminating any need for biasing the angle of attack loop. This also eliminated the need to make any of the aircraft parameters variable. Utilizing the system shown in Figure 13, the computer program was revised to put the aircraft into thirty degrees angle of bank at a rate of roll of ten degrees per second.

Figures 14a, 14b, and 14c show the response of the aircraft to entering this thirty degree angle of bank turn with no perturbations introduced to the airframe due to the turn. When the velocity perturbation was introduced alone, the response obtained was that shown in Figures 15a, 15b, and 15c. With both the velocity and angle of attack perturbations introduced, the response was that shown in Figures 16a, 16b, and 16c.

Of these three responses, the one with only the velocity perturbation introduced produced the results most closely agreeing with the desired steady state as shown in Figure 11. The response with no perturbations introduced shows the changes of velocity, angle of attack and thrust all tending toward zero after 90 seconds. When both velocity and angle of attack perturbations were used the response of the aircraft shows the velocity and angle of attack both tending toward zero while the thrust has decreased more than 9000 pounds and is still decreasing. The response of the aircraft

with only the velocity perturbation introduced shows the velocity steadying

at about 19 feet per second, the angle of attack approaching zero and the

thrust change steadying at about 1450 pounds. This is close to the desired

steady state with the undesired being that the thrust has steadied at a value

about 680 pounds too high.

5.   Conclusions and Recommendations.

During this investigation the use of the digital computer was found

very satisfactory in that originally putting the system into a useable program

was much easier than it would have been to wire an analog board. Also,

it was not necessary to do any scaling of the equations for the digital com-

puter, as it would have been for the analog computer. It was found to be

quite easy to make changes to the system when the digital program was used.

The difficult part of this investigation was in simulating the aircraft

making a turn. The perturbations introduced to the longitudinal equations

of motion by angle of bank are not obvious and a completely satisfactory

simulation of the turn was not realized in this investigation. A more thorough

investigation of the effect of angle of bank on the longitudinal equations of

motion is needed to reach a definite conclusion as to what changes must

be made in the basic automatic throttle system to handle a turn properly.

Another perturbation that possibly should have been included in this

investigation is the rate of change of pitch attitude. Appendix VI develops

a relationship between the pitch rate and the normal acceleration which is

itself a function of the angle of bank of the aircraft. Including this pertur-

bation may simulate the aircraft in a turn better than simulations included

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in this report. The perturbations are introduced to the airframe equations of motion in the following manner.

X-Force

$$\dot{u} = \frac{X_u}{m}(u_t + u) + \frac{X_w U_o}{m}(\alpha_t + \alpha) - g\theta + \frac{X_{\Delta T}}{m}\Delta T$$

Z-Force

$$\dot{\alpha} = \frac{Z_u}{mU_o}(u_t + u) + \frac{Z_w}{m}(\alpha_t + \alpha) + (\dot{\theta}_t + \dot{\theta}) + \frac{Z_\eta}{mU_o}\eta$$

Moment

$$\ddot{\theta} = \frac{M_u}{B}(u_t + u) + \frac{M_{\dot{w}}U_o}{B}\dot{\alpha} + \frac{M_w U_o}{B}(\alpha_t + \alpha) + \frac{M_q}{B}(\dot{\theta}_t + \dot{\theta})$$

$$+ \frac{M_\eta}{B}\eta + \frac{M_{\Delta T}}{B}\Delta T$$

It is recommended that any further investigations along these lines be continued as indicated in the preceding equations. It is further recommended that the possibility of using the hybrid analog-digital computer be investigated. This system would have the advantages of both computers and allow a great amount of versatility.

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5. Anon. Criterial for Predicting Landing Approach Speed Based on an Analog Computer Analysis of 21 Jet Propelled Aircraft. Vought Aeronautics. Report No. EOR - 13202, 1960

TABLE I

F-8 Aircraft Parameters

$U_O = V_a = 234$ ft/sec

$\bar{c} = 11.78$ ft

c.g. @ 24% $\bar{c}$

$W = 22,000$ lb

$m = 683.8$ slugs

$\dfrac{X_u}{m} = -0.06$ 1/sec

$\dfrac{X_w U_O}{m} = -3.30$ ft/sec^2

$\dfrac{X_{\Delta T}}{m} = 0.00145$ ft/lb-sec^2

$\dfrac{Z_u}{mU_O} = -0.00113$ 1/ft

$\dfrac{Z_w}{m} = -0.426$ 1/sec

$\dfrac{Z_\gamma}{mU_O} = -0.059$ 1/sec

$\dfrac{M'_u}{B} = 0.000189$ 1/ft-sec

$\dfrac{M_{\dot{w}} U_O}{B} = -0.041$ 1/sec

$\dfrac{M_w U_O}{B} = -1.14$ 1/sec^2

$\dfrac{M_q}{B} = -0.338$ 1/sec

$\dfrac{M_\gamma}{B} = -2.50$ 1/sec^2

$\dfrac{M_{\Delta T}}{B} = 4.55 \times 10^{-6}$ 1/lb-sec^2

Figure 1

Block Diagram of Automatic Throttle
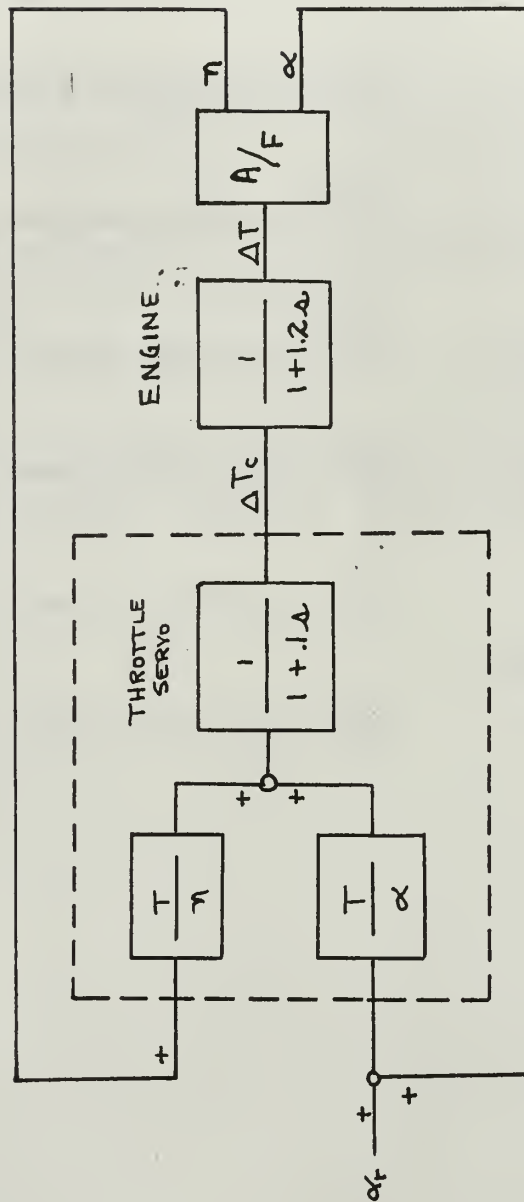
Airframe System

Figure 2

C_D versus C_L
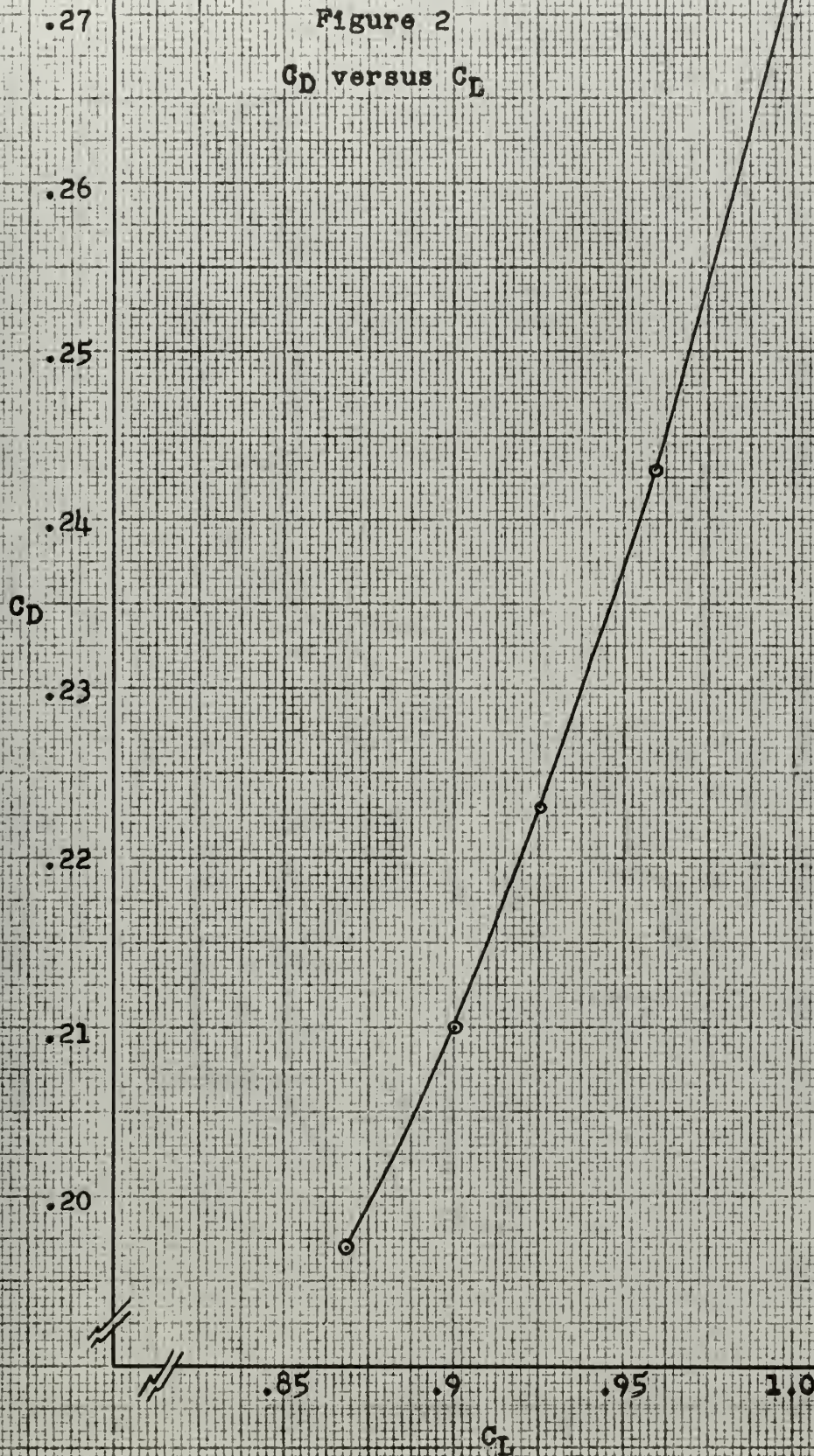
Figure 3

Flow Graph of Automatic Throttle Airframe System

FIGURE 4

COMPUTER PROGRAM OF AUTOMATIC
THROTTLE AIRFRAME SYSTEM

```
   PROGRAM FLYIT 2
   DIMENSION  X(30), XDOT(30), C(15)
   C(10) = 1.0
   PI=4.*ATANF(1.)
 1 CALL INTEG1     (T,X,XDOT,C)
```

THIS PROGRAM IS WITH AUTO-THROTTLE
AIRCRAFT ENTERING THIRTY DEGREE ANGLE
OF BANK TURN

 FOR LEVEL FLIGHT REMOVE 4 NEXT CARDS

```
   IF (T-3.) 2,3,3
 2 PHI=10.*T
   GO TO 4
 3 PHI=30.
 4 A=COSF(PHI*PI/180.)
   XU=-.06
   XW=-3.3
   ZU=-.00113
   ZW=-.426
   ELEV=(.381/.834)*X(2)
   GEE=1./A-1.+(234./32.2)*(X(4)-X(15))
   XDOT(1)=XU*X(1)-32.2*X(3)+XW*X(2)+.00145*X(5)
   XDOT(2)=ELEV*(-.059)+X(4)+ZW*X(2)+ZU*X(1)
   X(15)=XDOT(2)
   XDOT(3)=X(4)
   XDOT(4)=ELEV*(-2.5)-.00000455*X(5)-.338*X(4)-
  1.041*X(15)-1.14*X(2)+.000189*X(1)
   XDOT(5)=(1./1.2)*(X(8)+X(6)-X(5))
   XDOT(6)=X(7)
   XDOT(7)=-10.*X(6)-11.*X(7)-194000.*GEE
   XDOT(8)=10.*(-X(8)+X(11)+X(10)+X(9))
   X(13)=X(2)
   XDOT(9)=-X(9)+1100.*X(13)*57.3
   XDOT(10)=190.*X(13)*57.3
   XDOT(11)=-2.*X(11)+1140.*X(13)*57.3
   GO TO 1
   END
   END
```

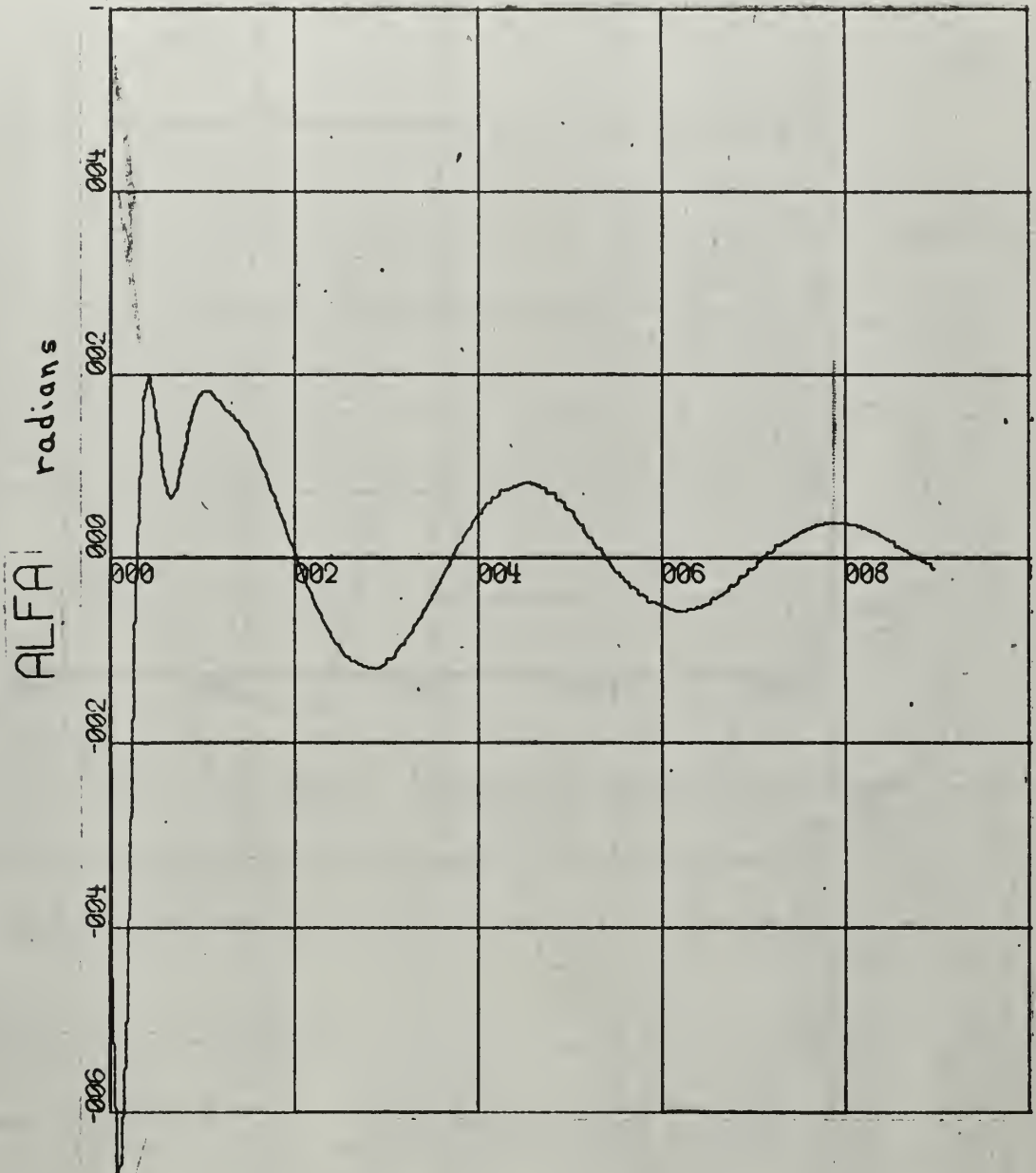DATA CARDS MUST BE ADDED TO RUN THIS PROGRAM
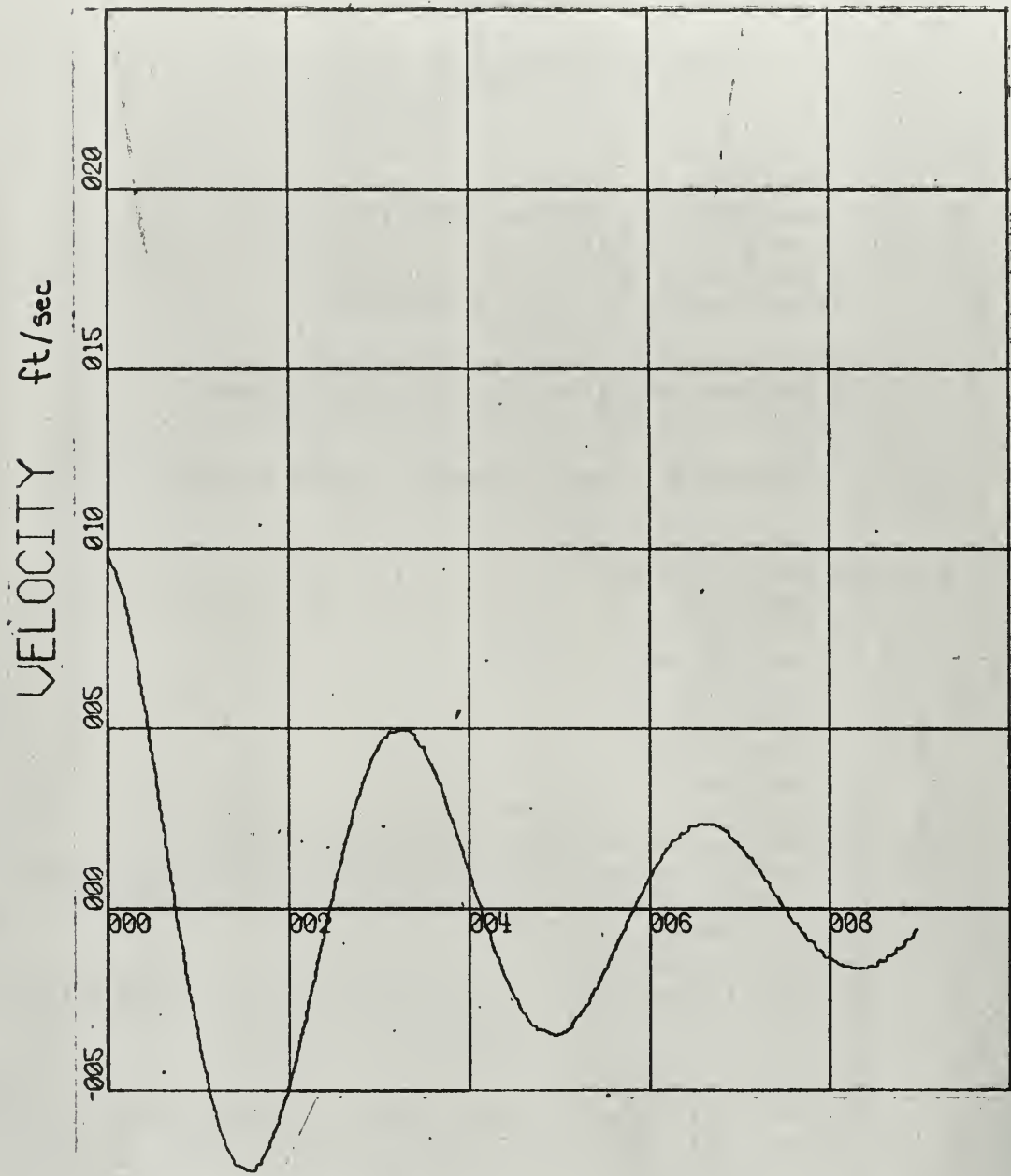
X-SCALE = 2.00E+01 UNITS/INCH.
Y-SCALE = 5.00E+00 UNITS/INCH.
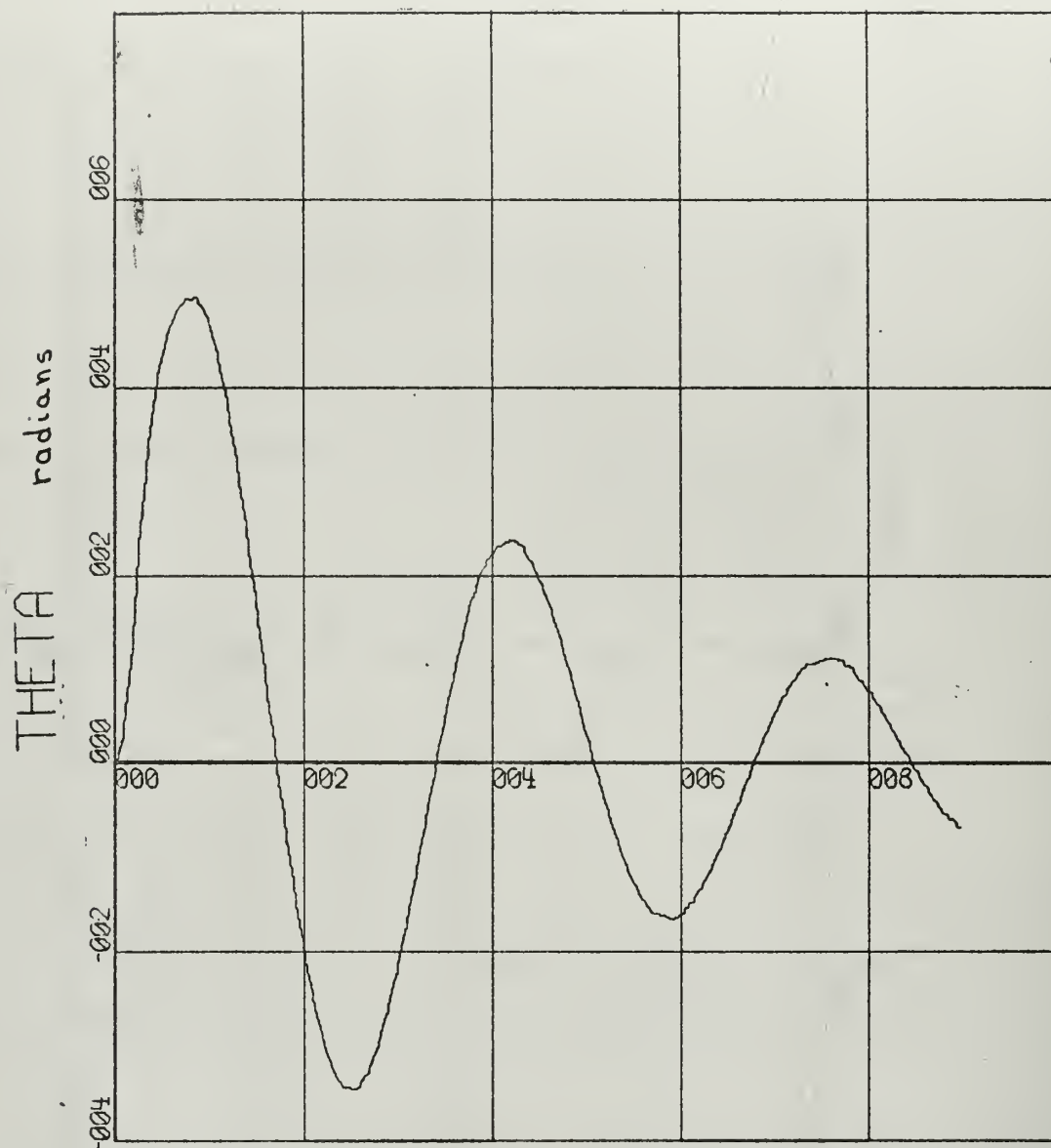
(a)

Figure 5

Aircraft Response to Initial Velocity Gust

24

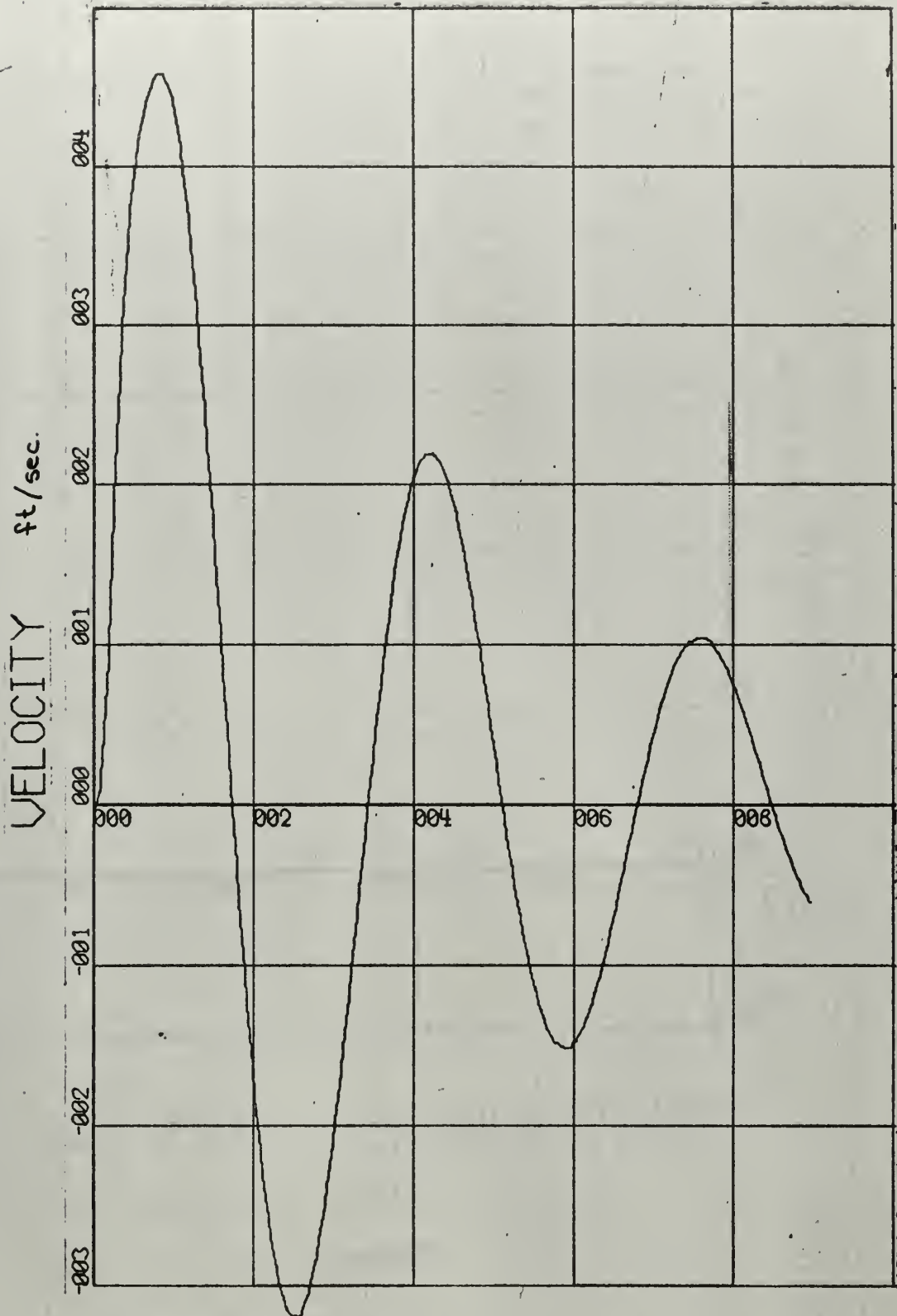X-SCALE = 2.00E+01 UNITS/INCH.
Y-SCALE = 2.00E-03 UNITS/INCH.

TIME   sec.

(b)

Figure 5

25

X-SCALE = 2.00E+01 UNITS/INCH.
Y-SCALE = 2.00E-02 UNITS/INCH.

TIME   sec.

(c)

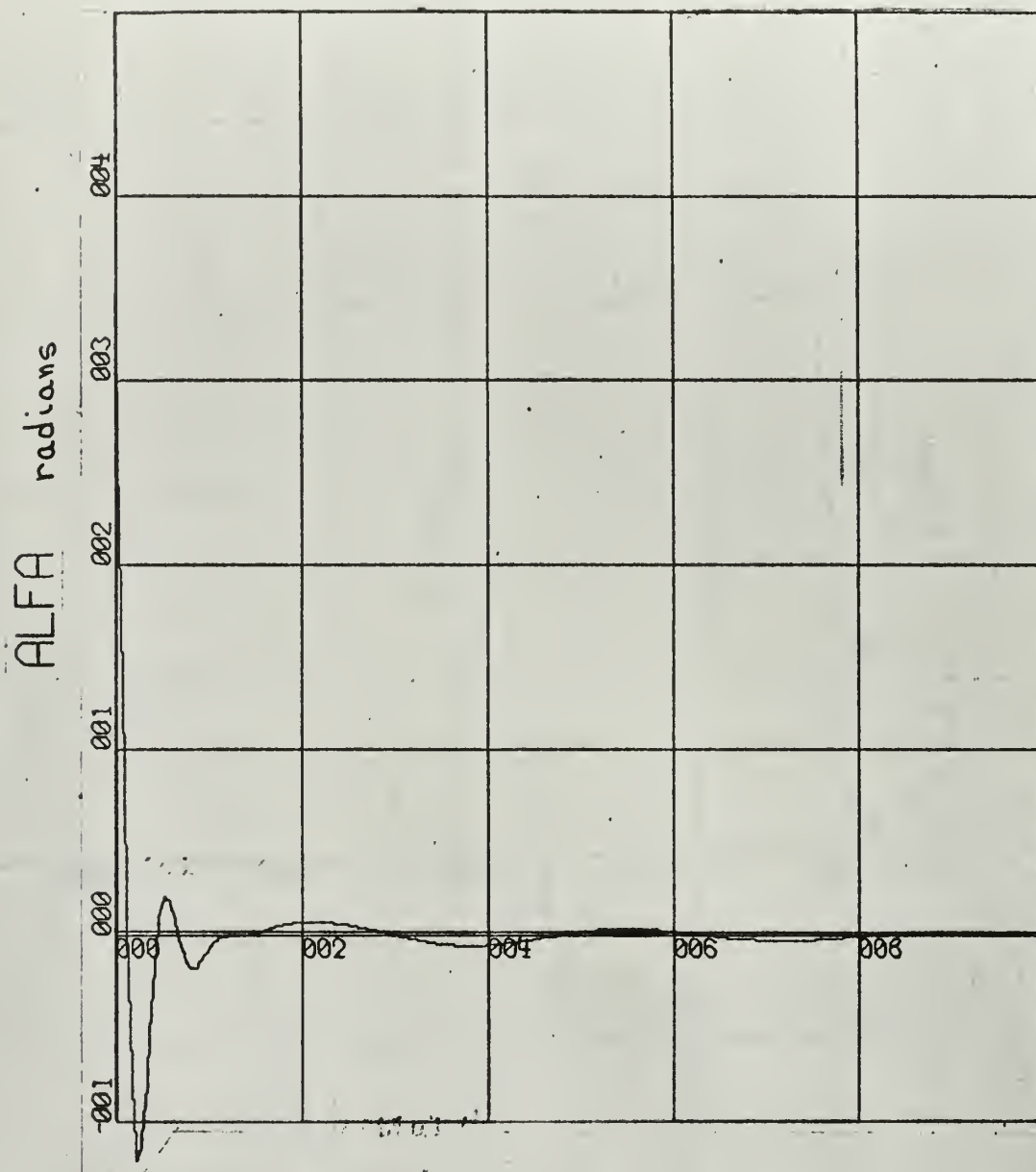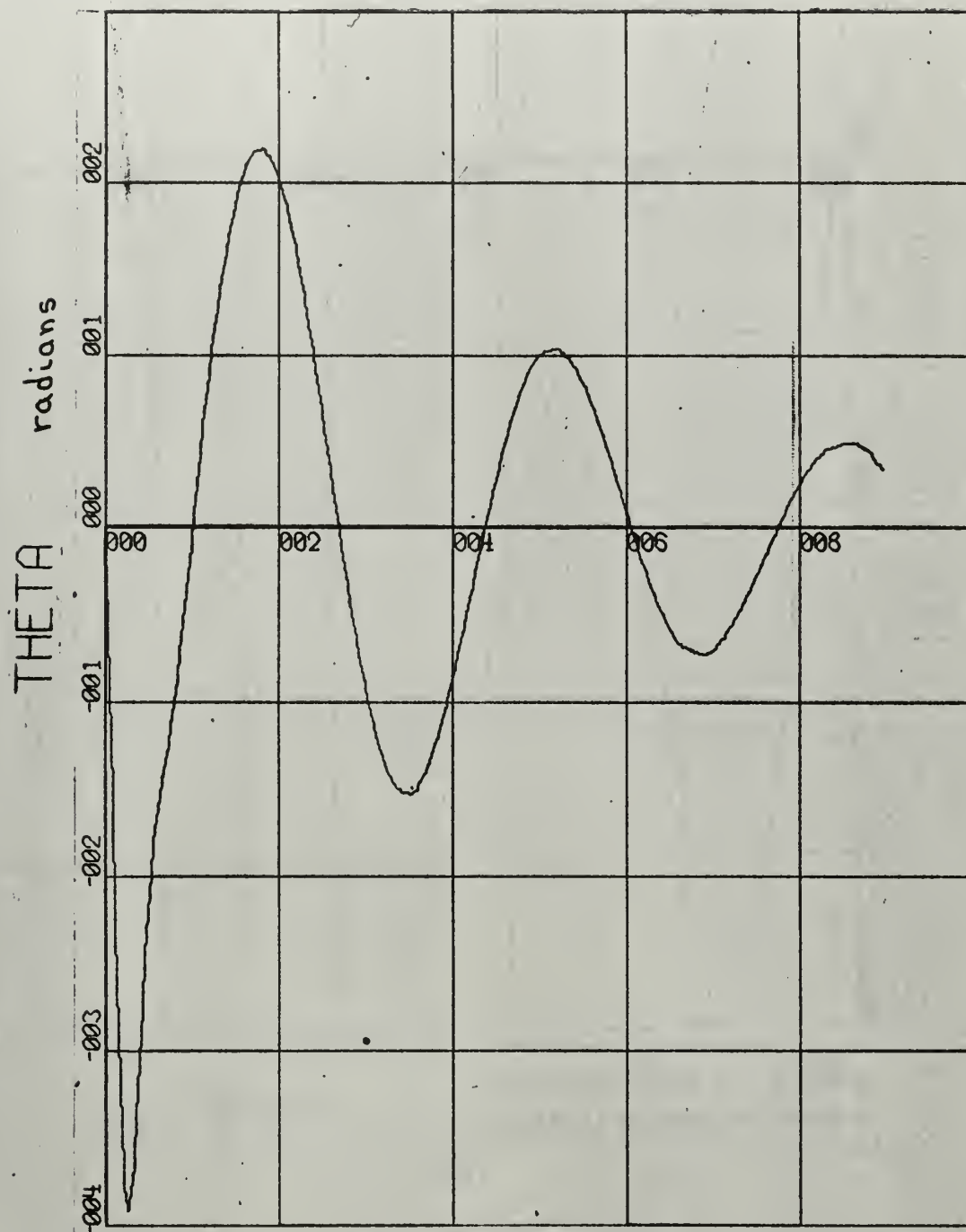Figure 5

VELOCITY ft/sec.

TIME sec.

X-SCALE = 2.00E+01 UNITS/INCH.
Y-SCALE = 1.00E+00 UNITS/INCH.

(a)

Figure 6

Aircraft Response to Initial Angle of Attack

X-SCALE = 2.00E+01 UNITS/INCH.
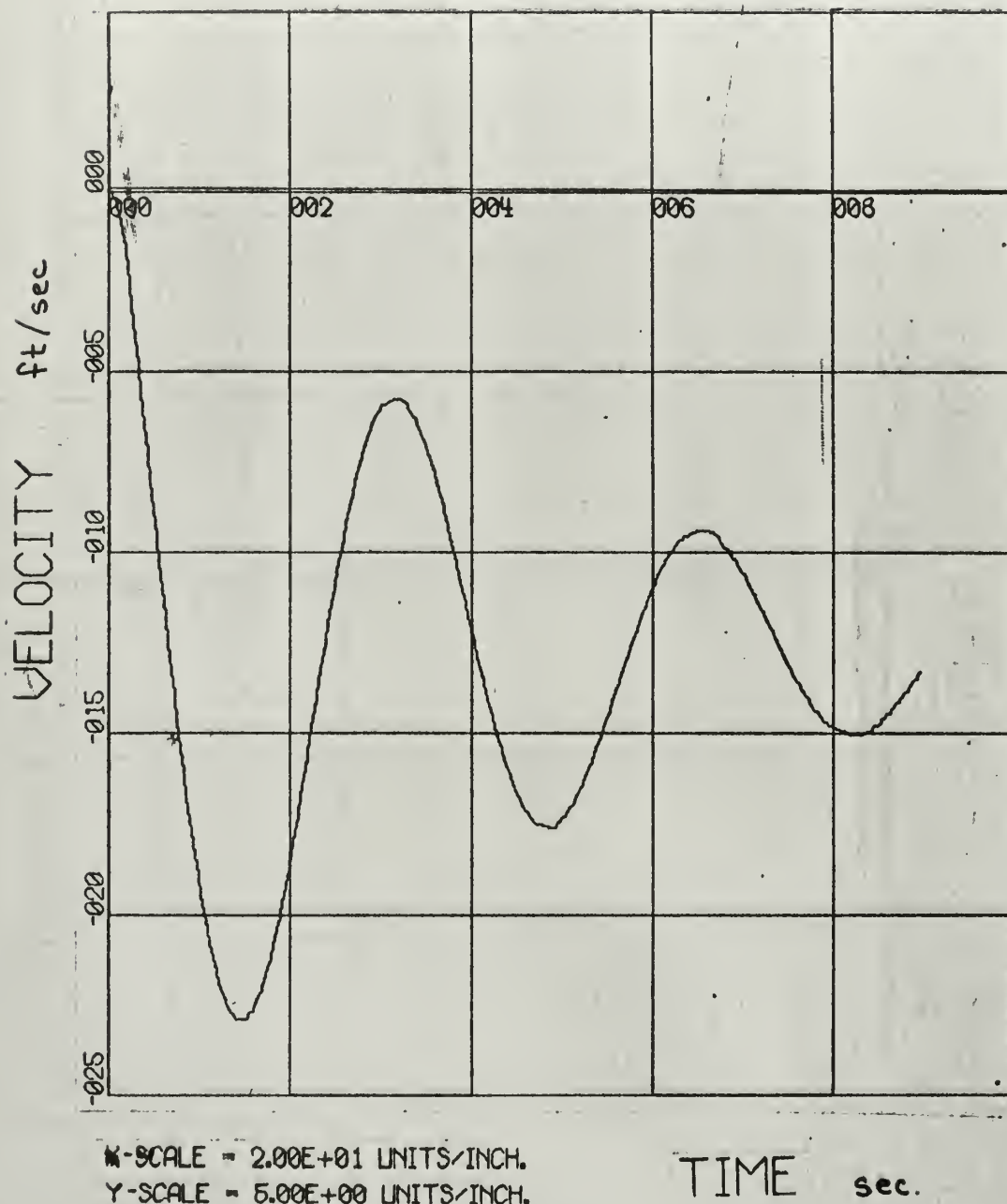Y-SCALE = 1.00E-02 UNITS/INCH.

TIME sec.

(b)

Figure 6

X-SCALE = 2.00E+01 UNITS/INCH.
Y-SCALE = 1.00E-02 UNITS/INCH.

TIME   sec.

(c)

Figure 6

29

X-SCALE = 2.00E+01 UNITS/INCH.
Y-SCALE = 5.00E+00 UNITS/INCH.

(a)

Figure 7

Aircraft Response to a Step
Input of Elevator

30

X-SCALE = 2.00E+01 UNITS/INCH.
Y-SCALE = 1.00E-02 UNITS/INCH.

(b)

Figure 7

31

X-SCALE = 2.00E+01 UNITS/INCH.
Y-SCALE = 2.00E-02 UNITS/INCH.

TIME   sec.

(c)

Figure 7

32

X-SCALE = 2.00E+01 UNITS/INCH.
Y-SCALE = 1.00E-02 UNITS/INCH.

TIME   sec.
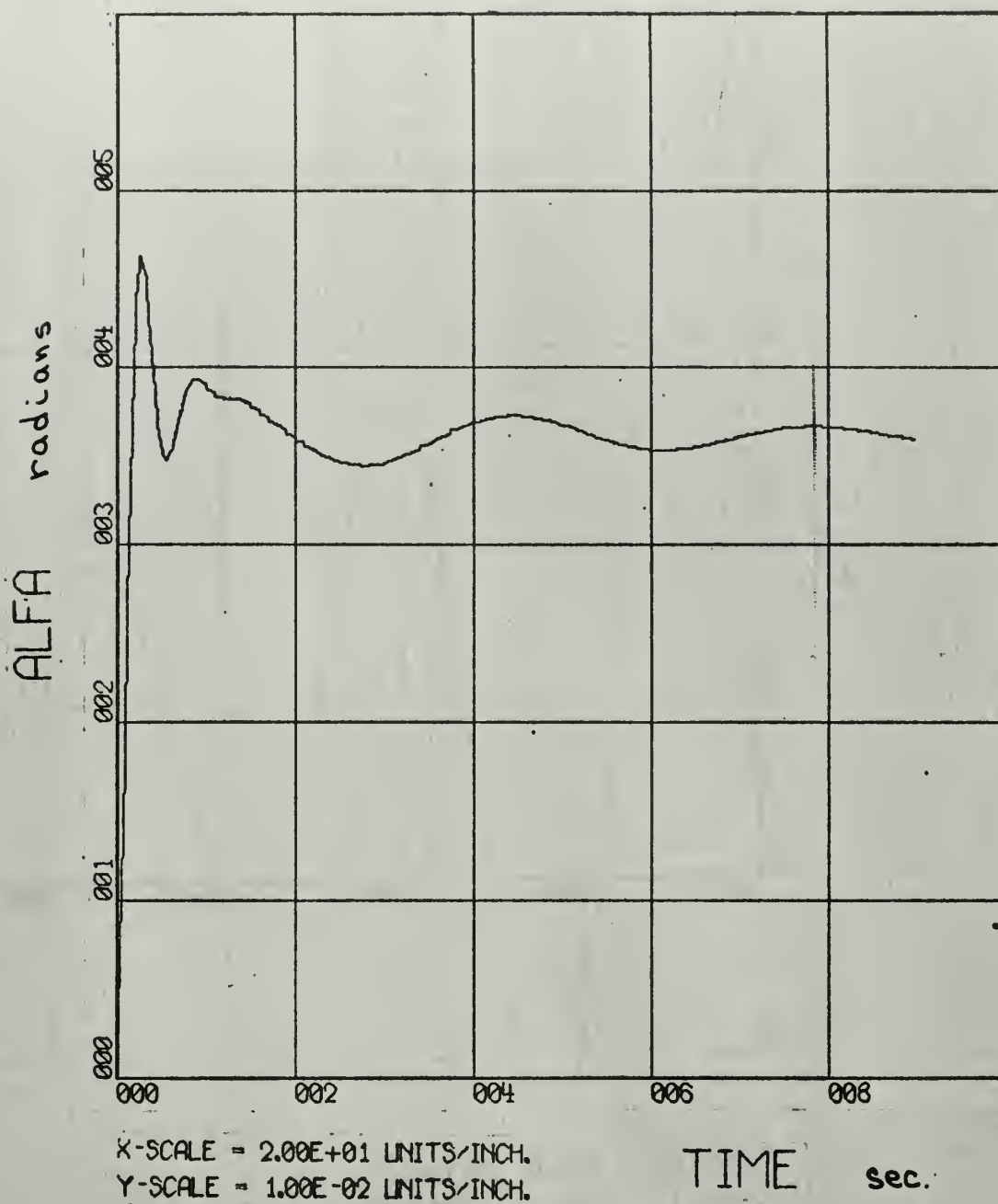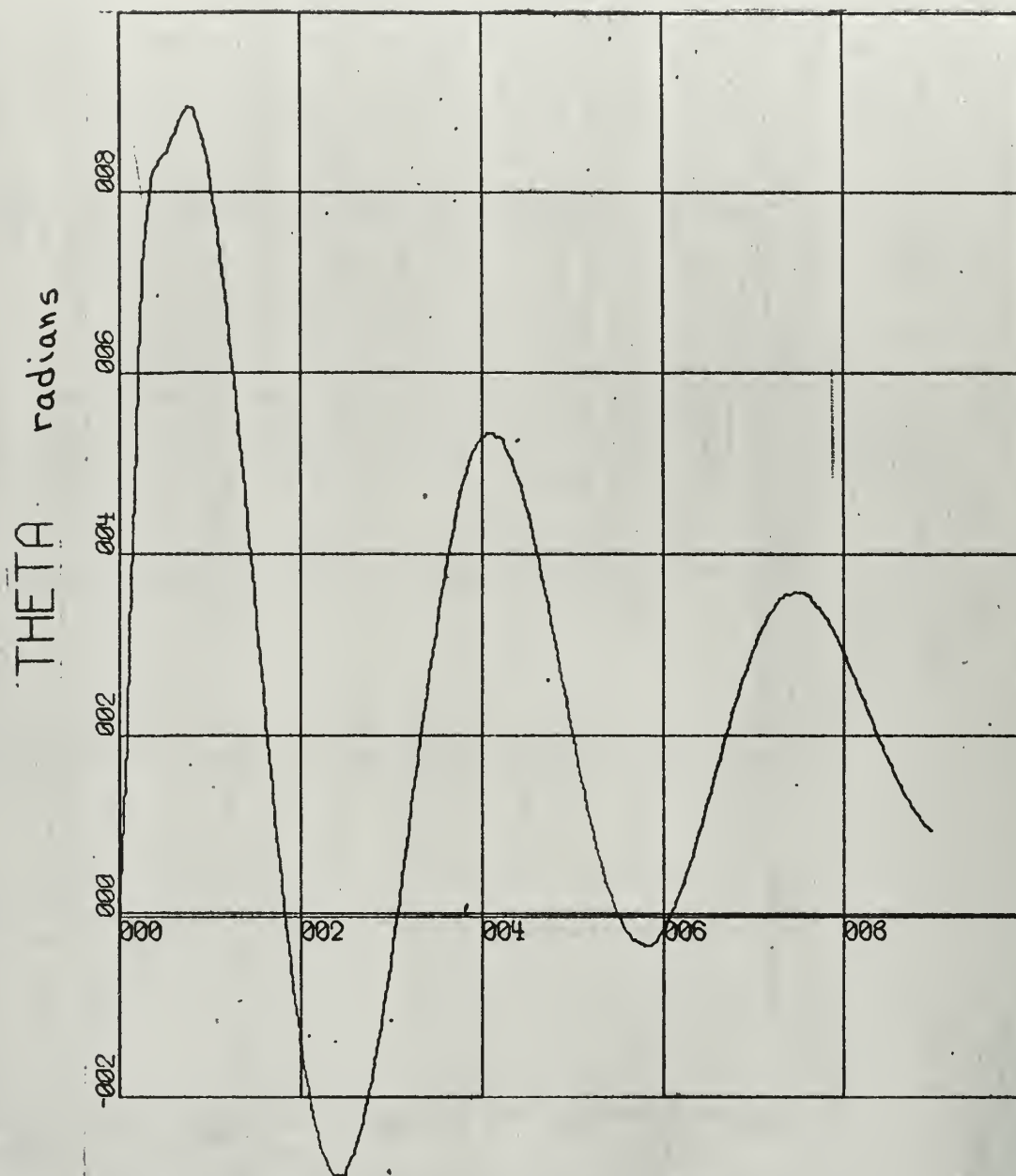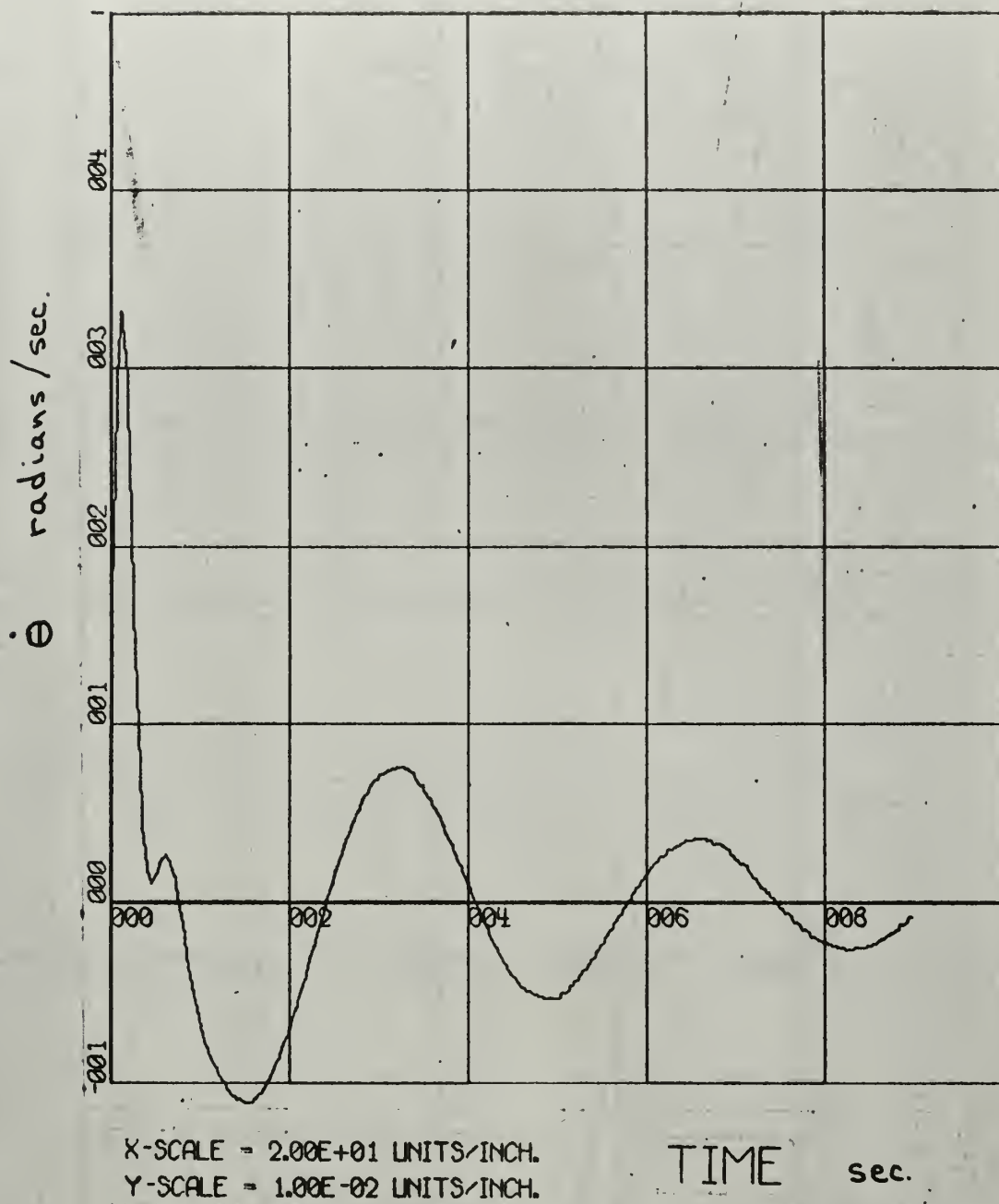
(d)

Figure 7

33

VELOCITY ft/sec.

TIME sec.

X-SCALE = 5.00E+00 UNITS/INCH.
Y-SCALE = 2.00E+00 UNITS/INCH.

(a)
Figure 8
Aircraft-Automatic Throttle Response to
Initial Velocity Gust (No Elevator)

34

K-SCALE = 5.00E+00 UNITS/INCH.
Y-SCALE = 2.00E-03 UNITS/INCH.

(b)

Figure 8

35

X-SCALE = 5.00E+00 UNITS/INCH.
Y-SCALE = 5.00E+02 UNITS/INCH.
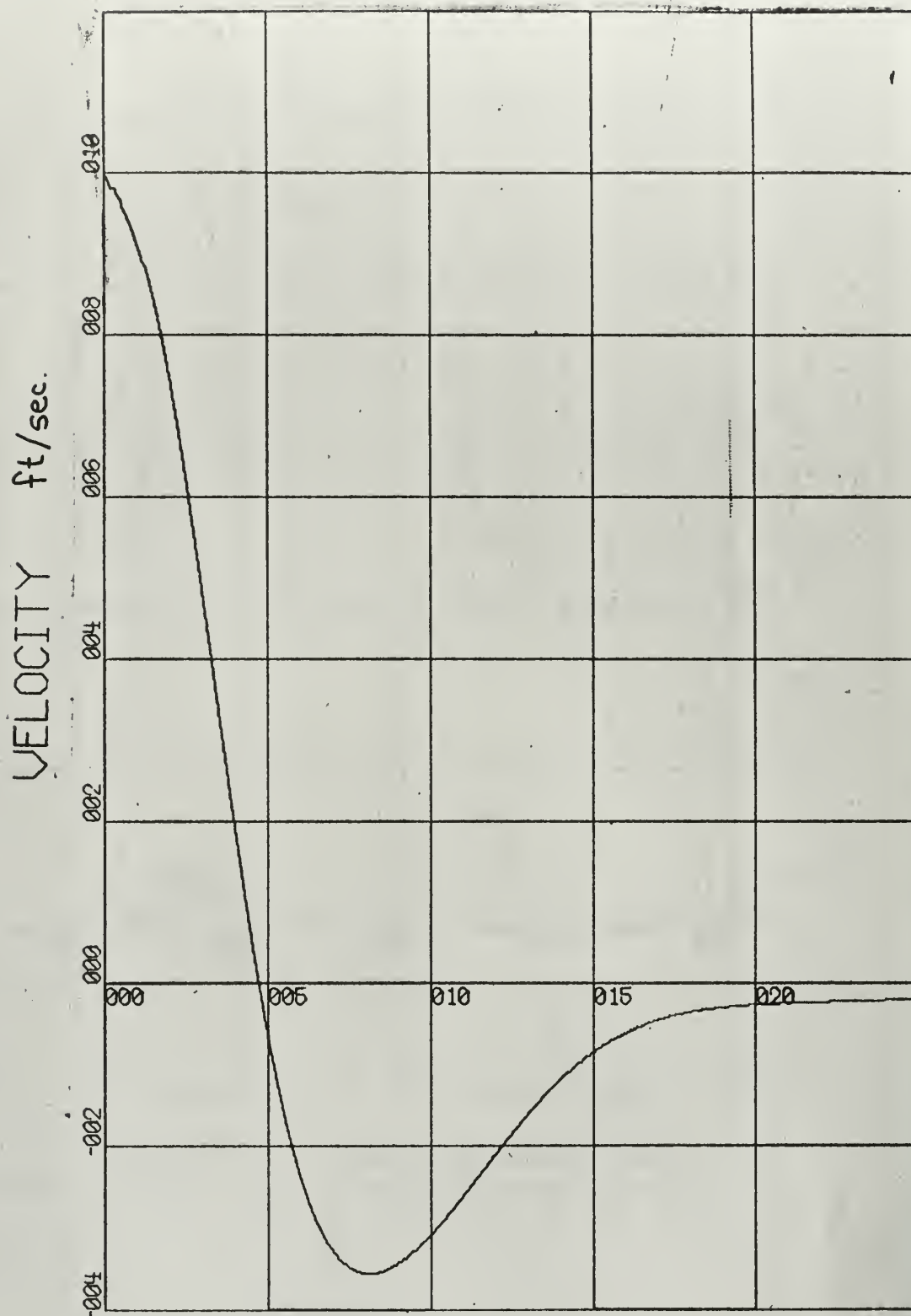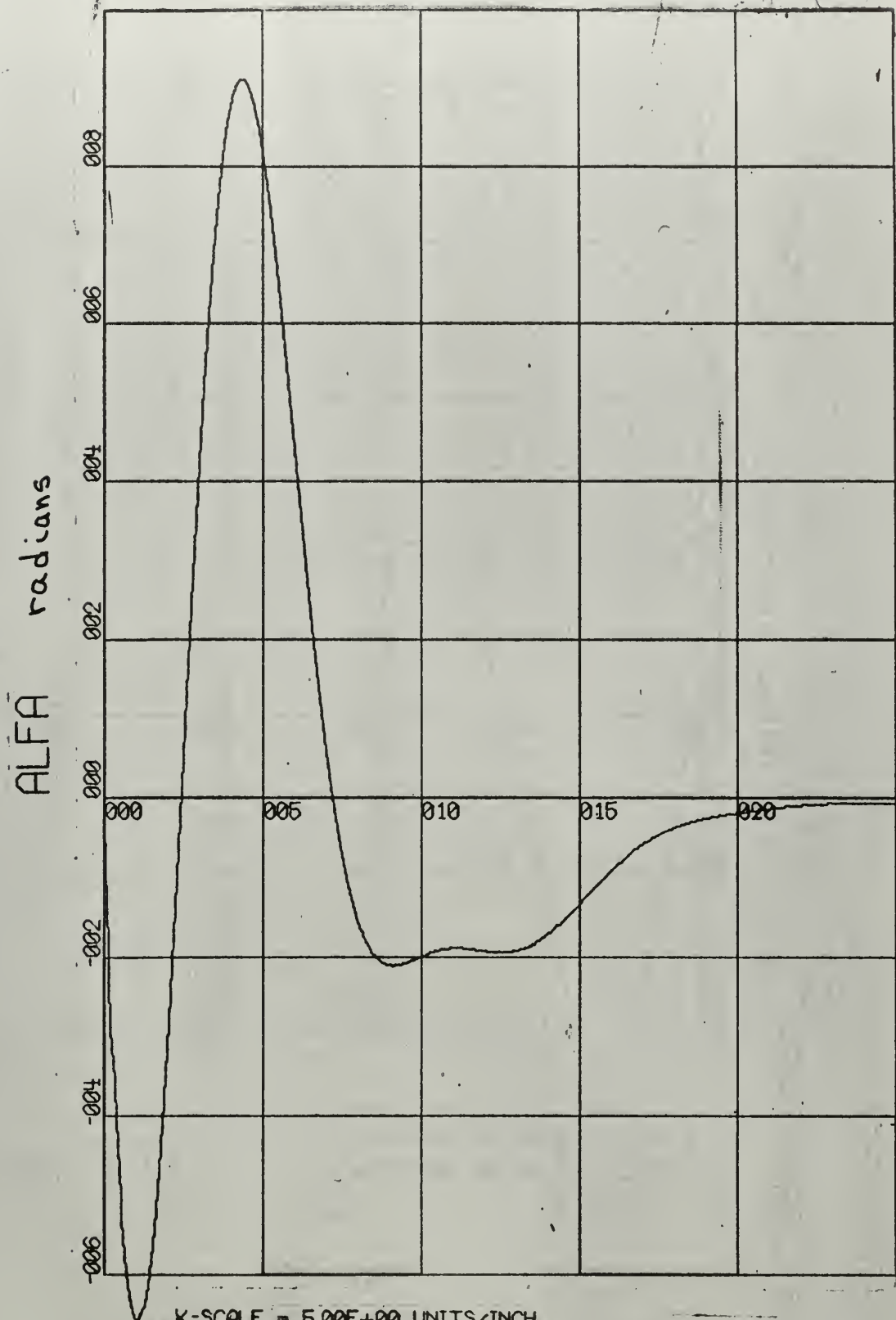
(c)

Figure 8

36

X-SCALE = 5.00E+00 UNITS/INCH.
Y-SCALE = 2.00E+00 UNITS/INCH.

(a)
Figure 9
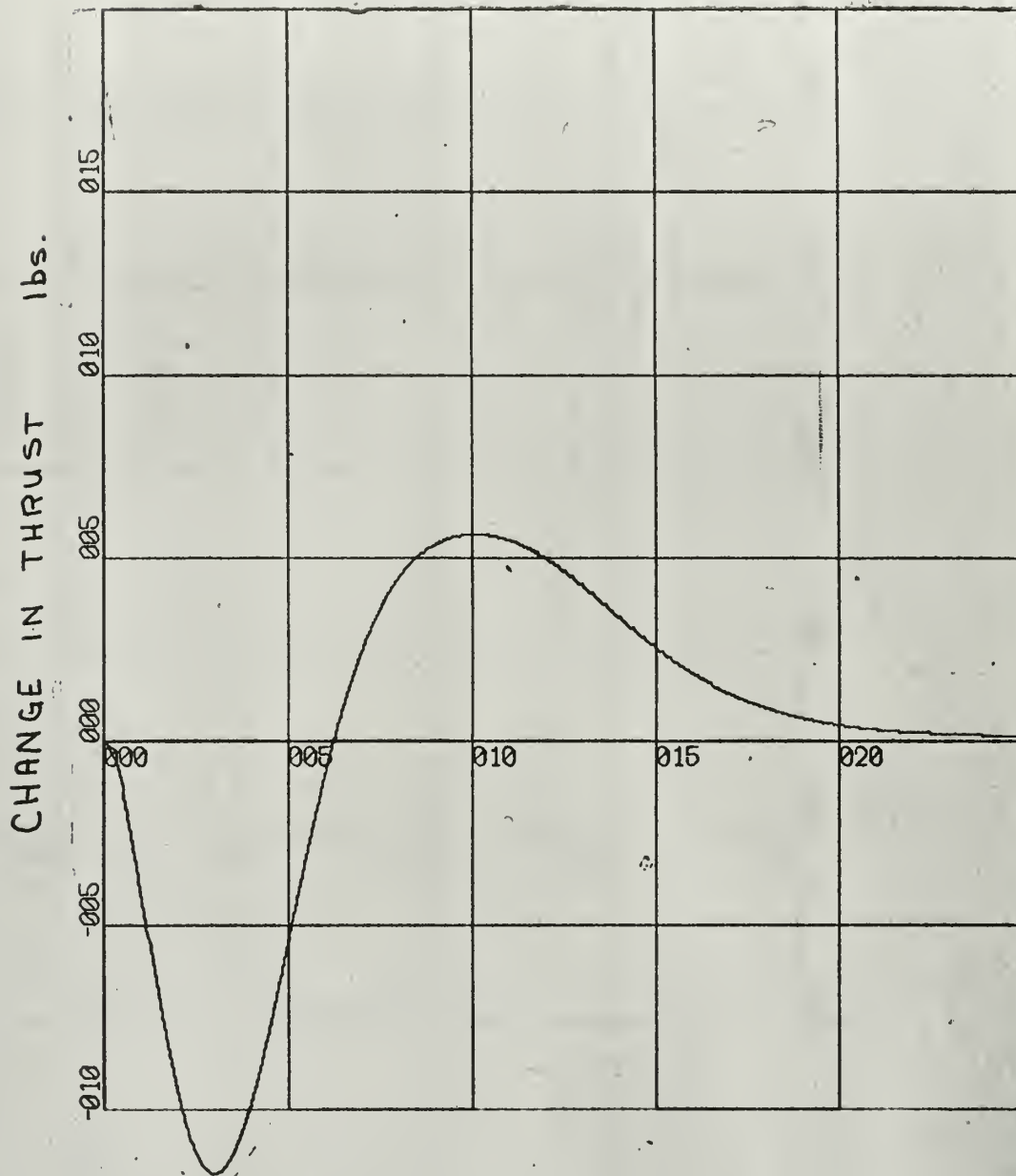Aircraft-Automatic Throttle Response to
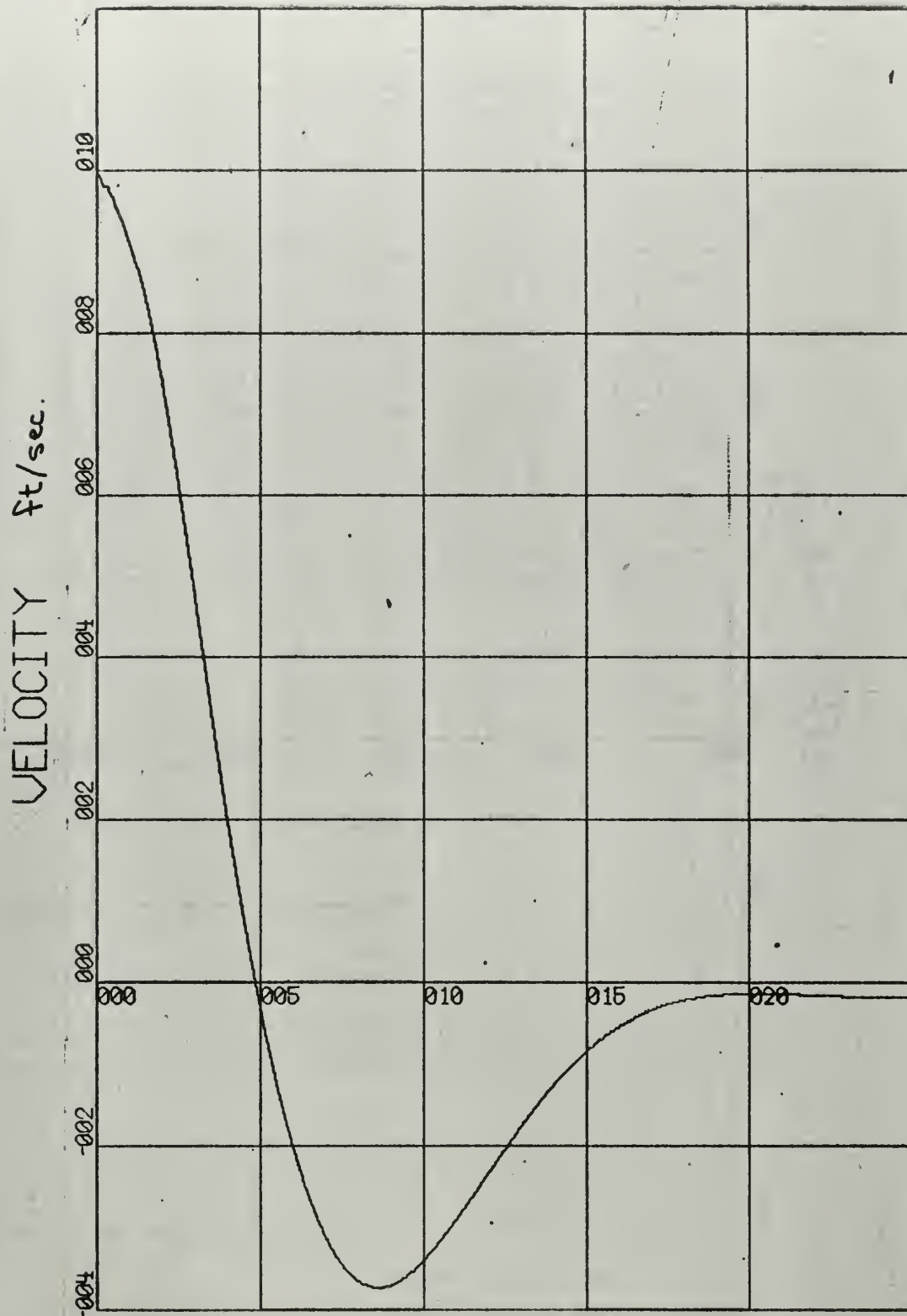Initial Velocity Gust (Elevator as a
Function of Angle of Attack)

X-SCALE = 5.00E+00 UNITS/INCH.
Y-SCALE = 2.00E-03 UNITS/INCH.

TIME sec.

(b)

Figure 9

38

X-SCALE = 5.00E+00 UNITS/INCH.
Y-SCALE = 2.00E+02 UNITS/INCH.

(c)
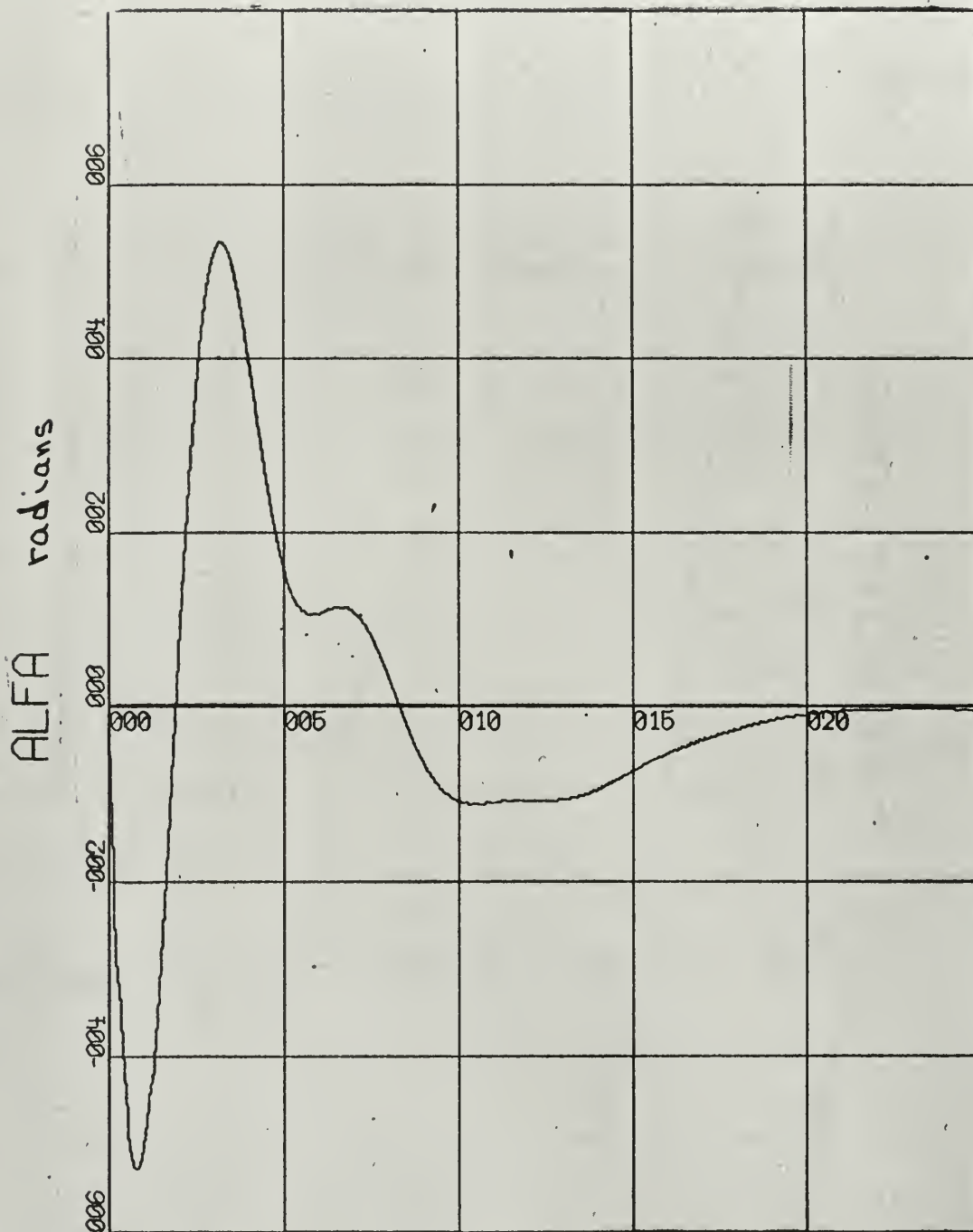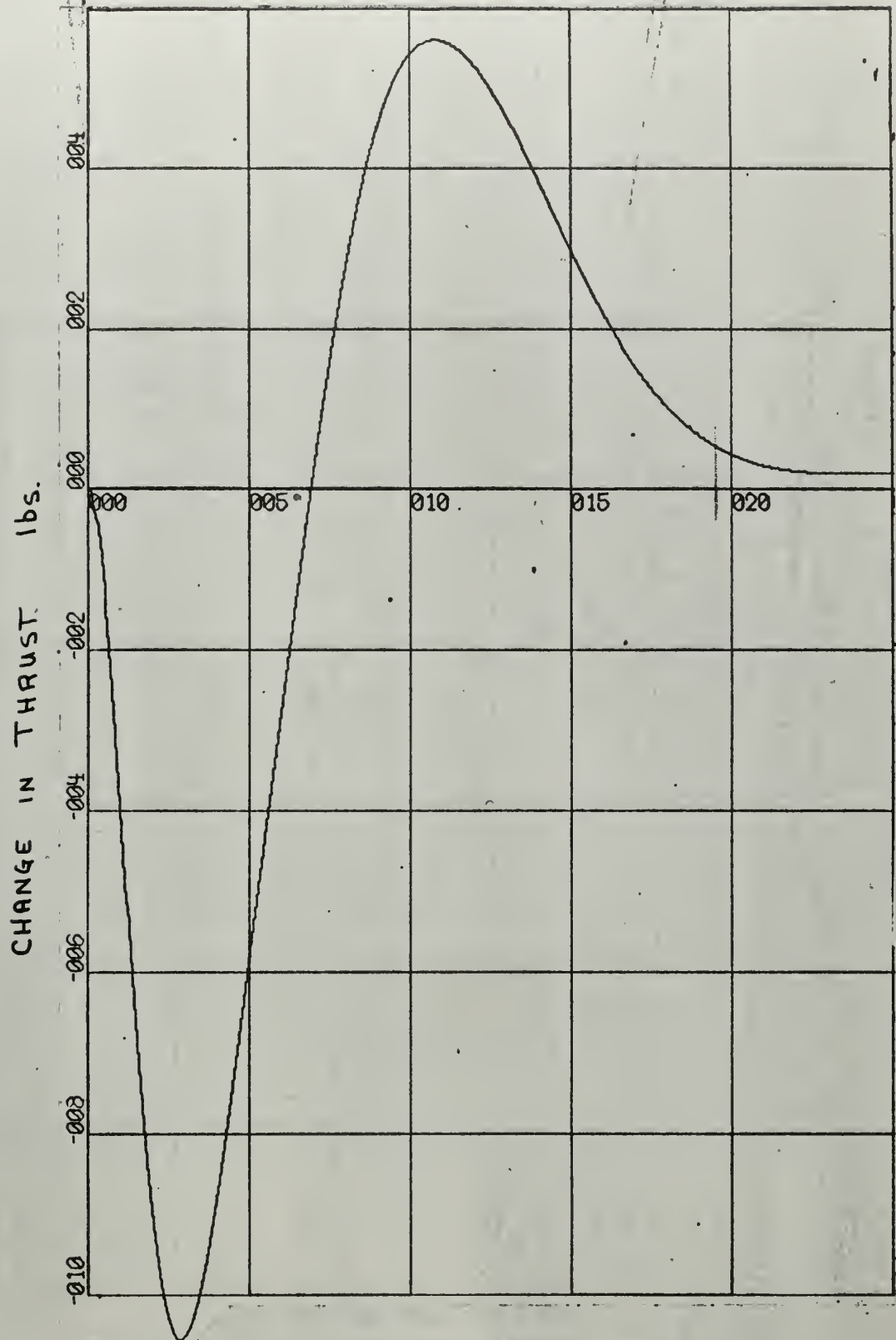
Figure 9

X-SCALE = 5.00E+00 UNITS/INCH.
Y-SCALE = 5.00E-01 UNITS/INCH.
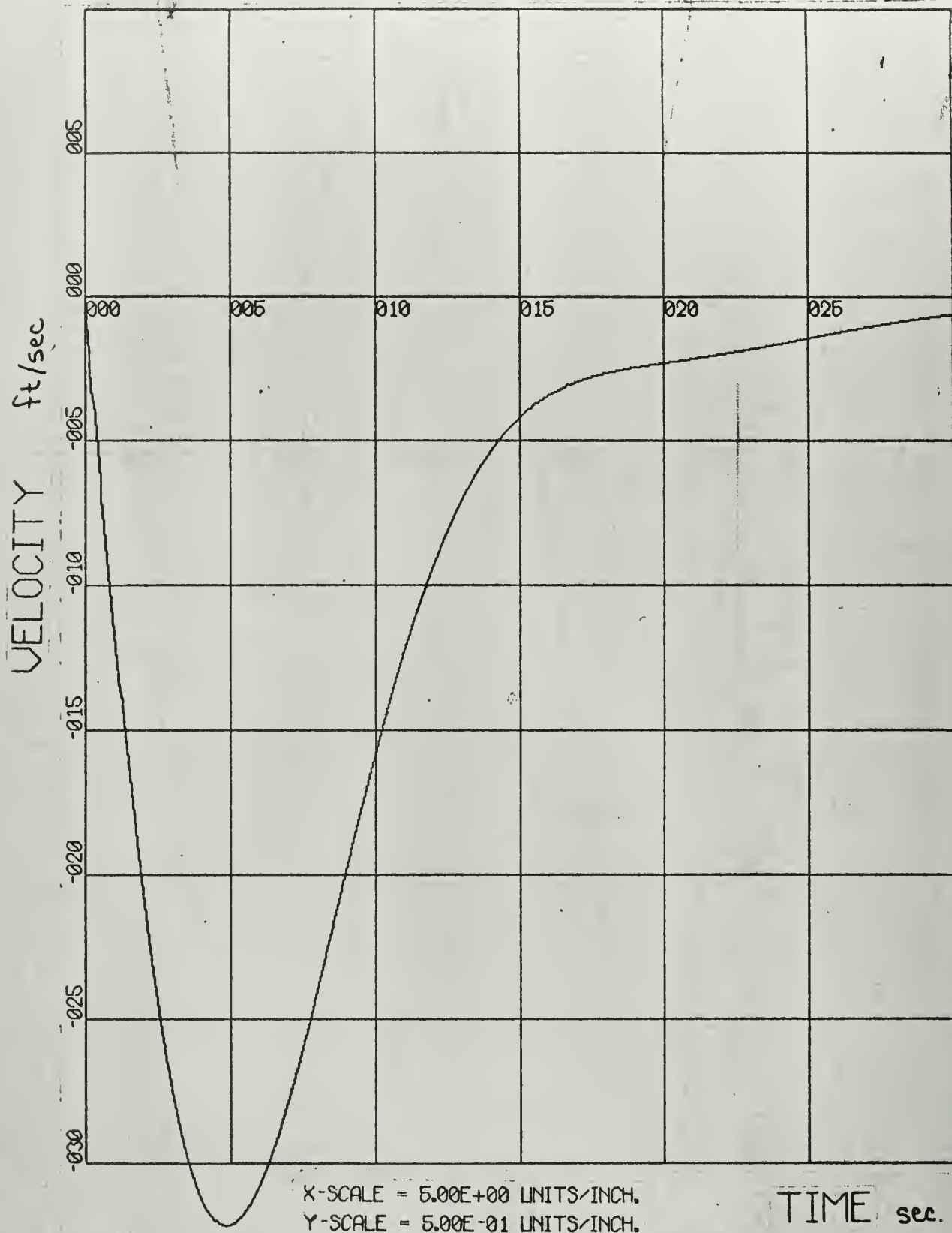
(a)
Figure 10
Aircraft-Automatic Throttle Response to
Initial Pitch Attitude

40

X-SCALE = 5.00E+00 UNITS/INCH.
Y-SCALE = 2.00E-04 UNITS/INCH.

(b)

Figure 10

41

X-SCALE = 5.00E+00 UNITS/INCH.
Y-SCALE = 1.00E+02 UNITS/INCH.

TIME sec.

(c)

Figure 10

42
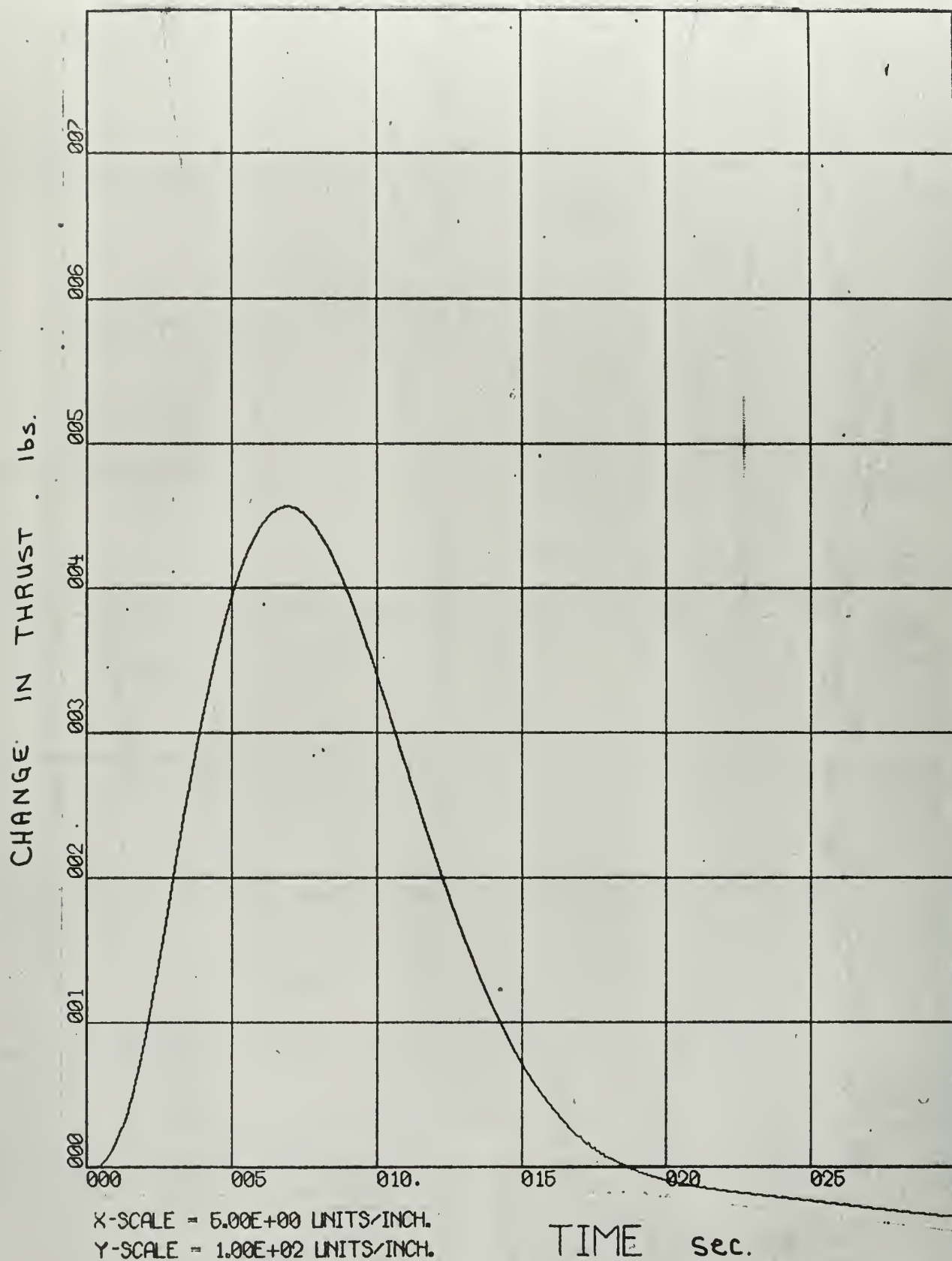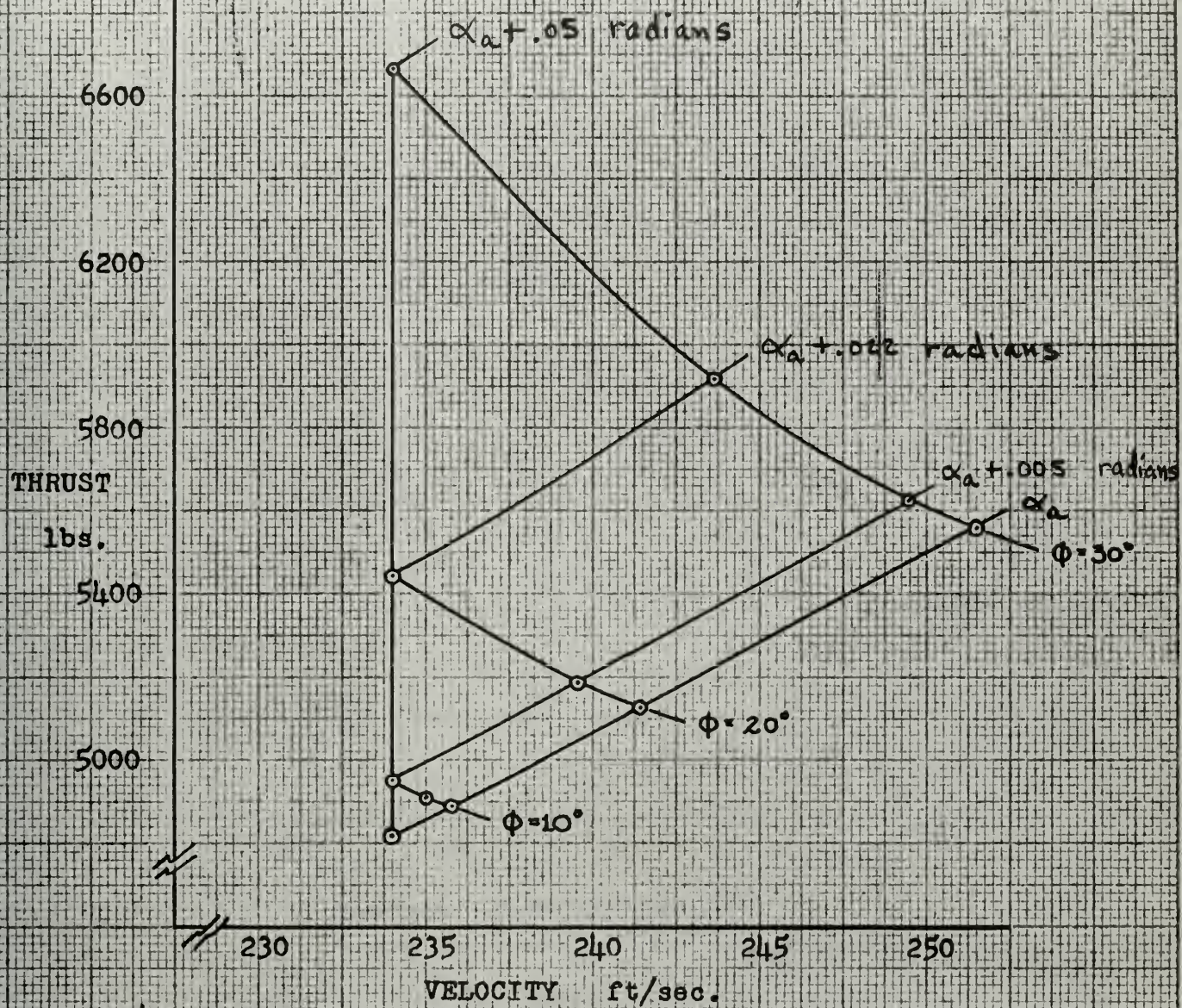
Figure 11

Thrust Required versus

Velocity

CHANGE

IN

THRUST

lbs.

ANGLE OF BANK degrees
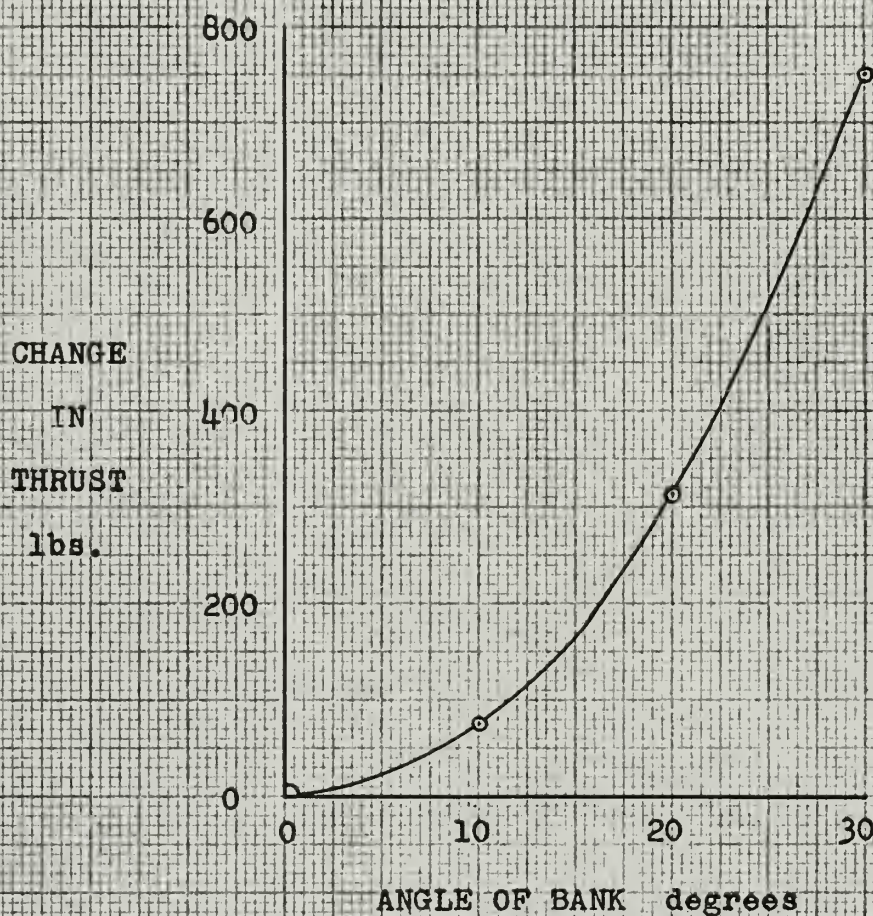
Figure 12

Change in Thrust

versus

Angle of Bank

(Constant Angle of Attack)

44

Figure 13

Block Diagram of Revised Automatic
Throttle Airframe System

X-SCALE = 2.00E+01 UNITS/INCH.
Y-SCALE = 5.00E-01 UNITS/INCH.
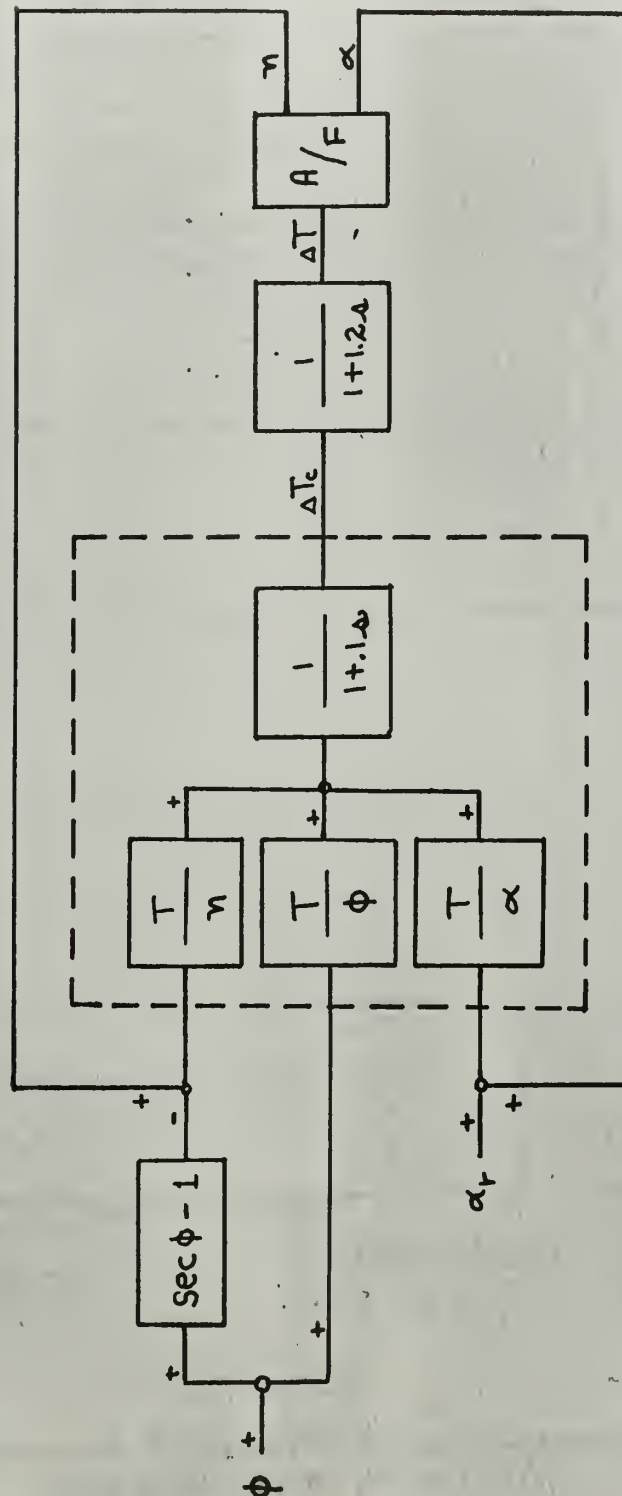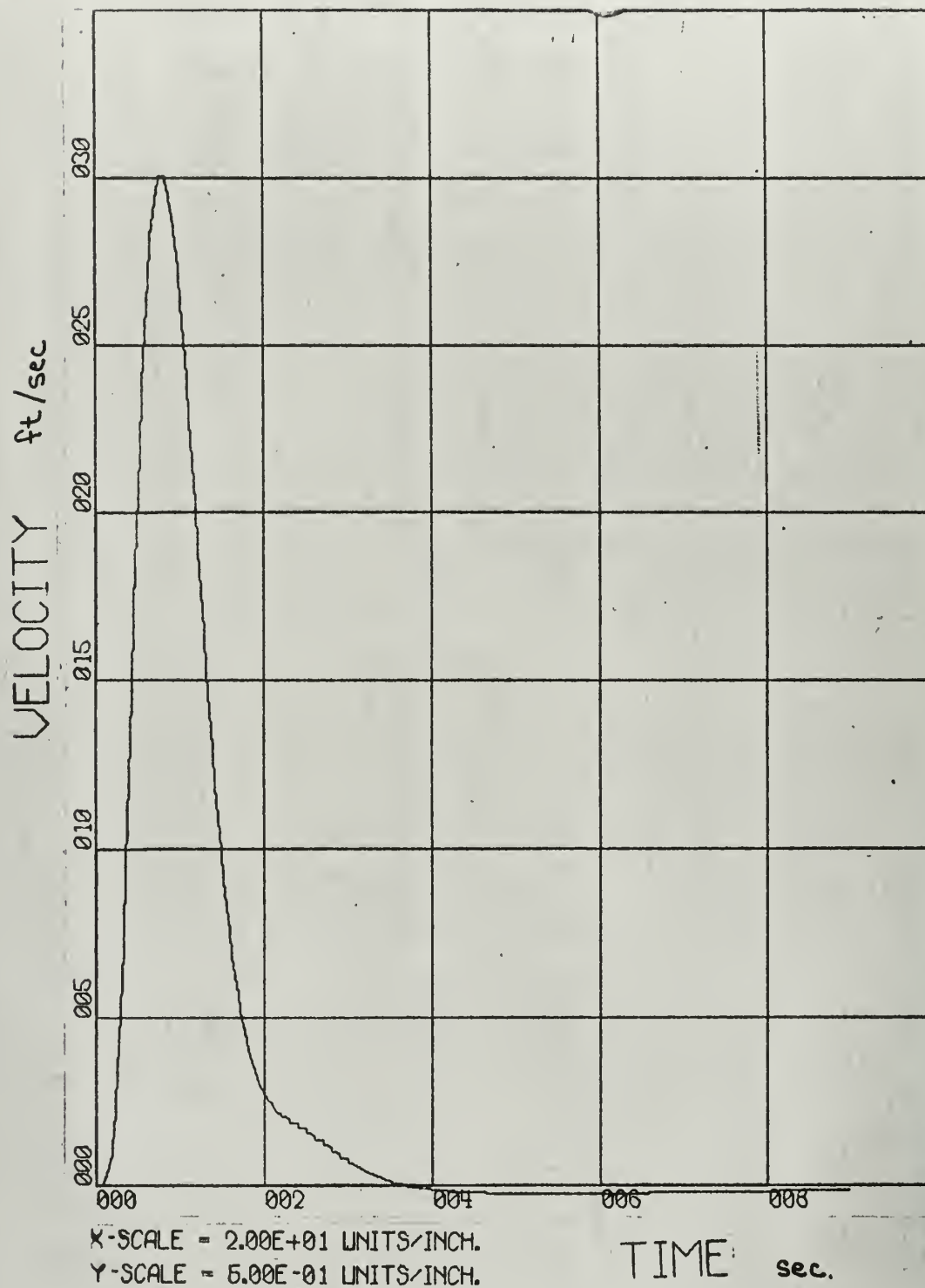
(a)
Figure 14

Aircraft-Automatic Throttle Response to
Turn with no Perturbations

46

X-SCALE = 2.00E+01 UNITS/INCH.
Y-SCALE = 5.00E-04 UNITS/INCH.

(b)

Figure 14

X-SCALE = 2.00E+01 UNITS/INCH.
Y-SCALE = 1.00E+02 UNITS/INCH.

(c)

Figure 14

48

X-SCALE = 2.00E+01 UNITS/INCH.
Y-SCALE = 5.00E+00 UNITS/INCH.
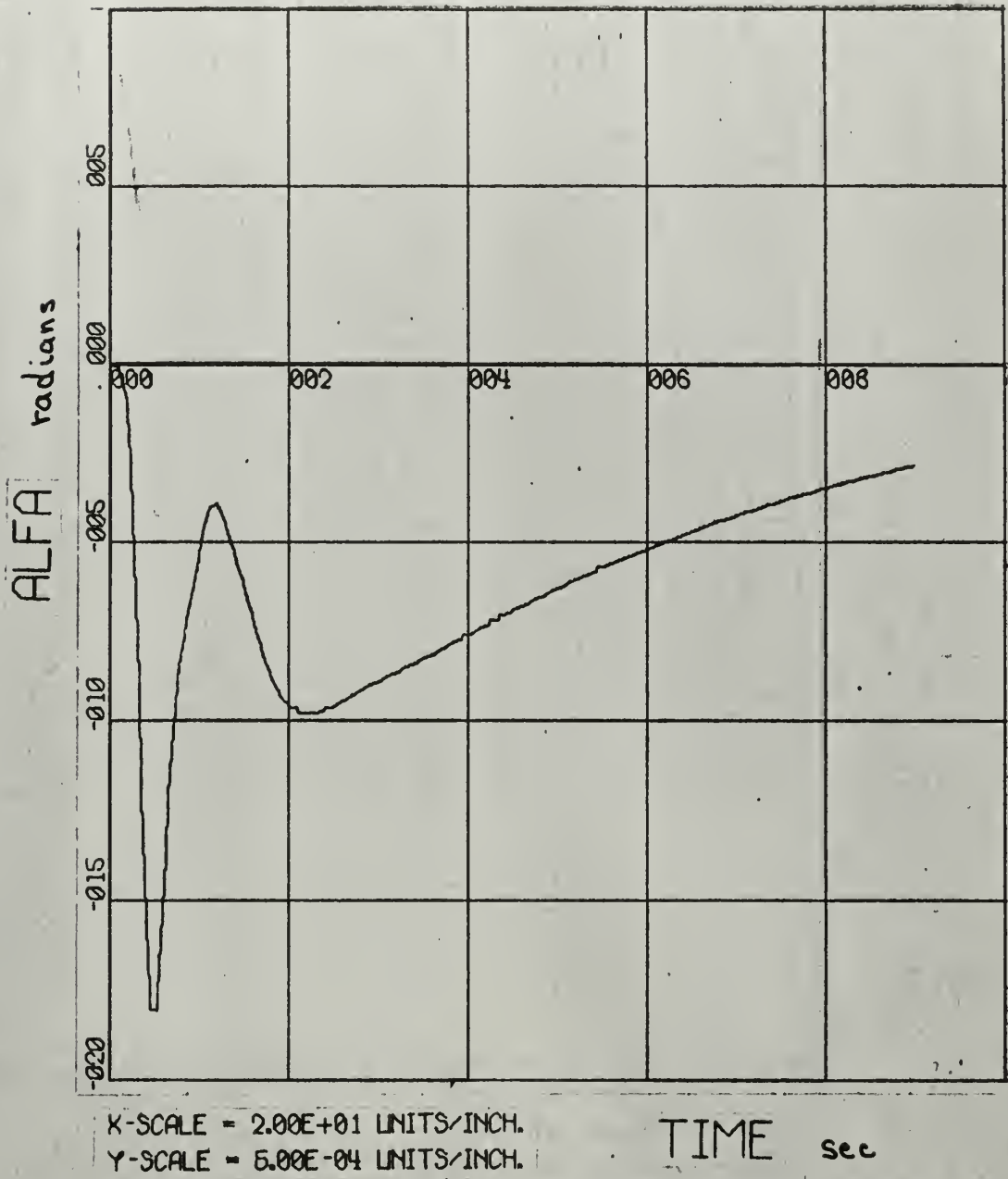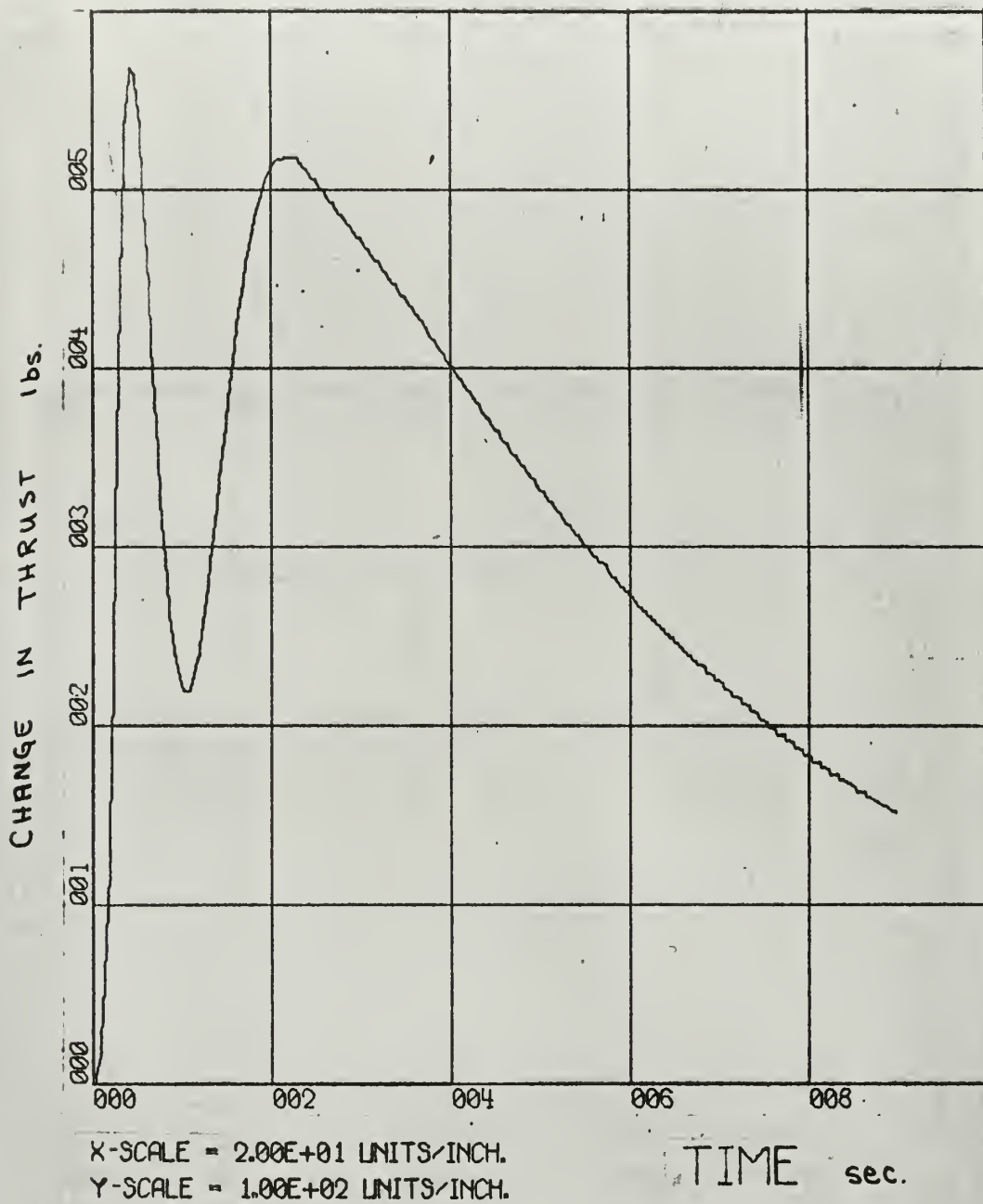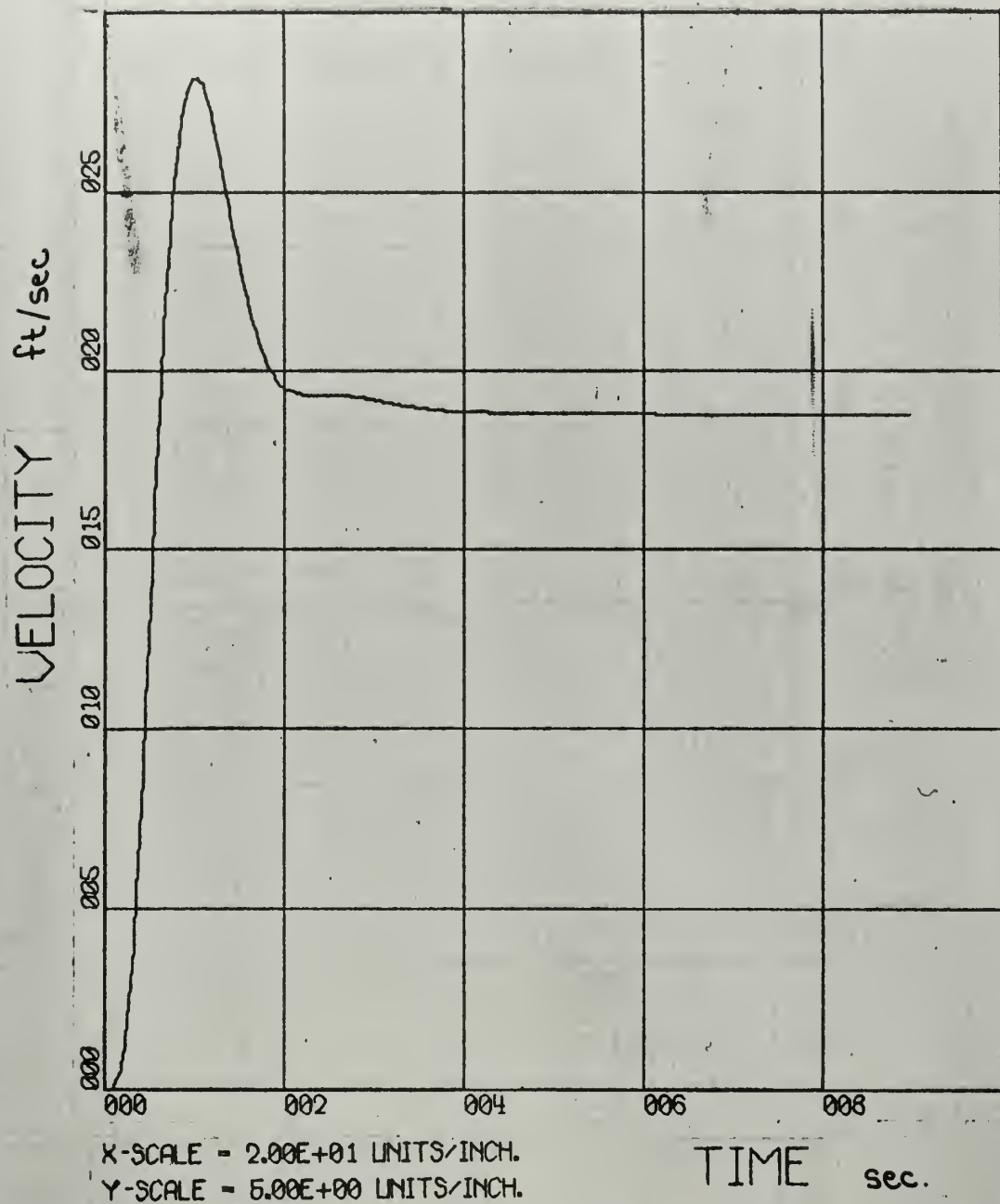
(a)

Figure 15

Aircraft-Automatic Throttle Response to
Turn with Velocity Perturbation

49

X-SCALE = 2.00E+01 UNITS/INCH.
Y-SCALE = 2.00E-03 UNITS/INCH.

TIME sec.

(b)

Figure 15

50

X-SCALE = 2.00E+01 UNITS/INCH.
Y-SCALE = 5.00E+02 UNITS/INCH.
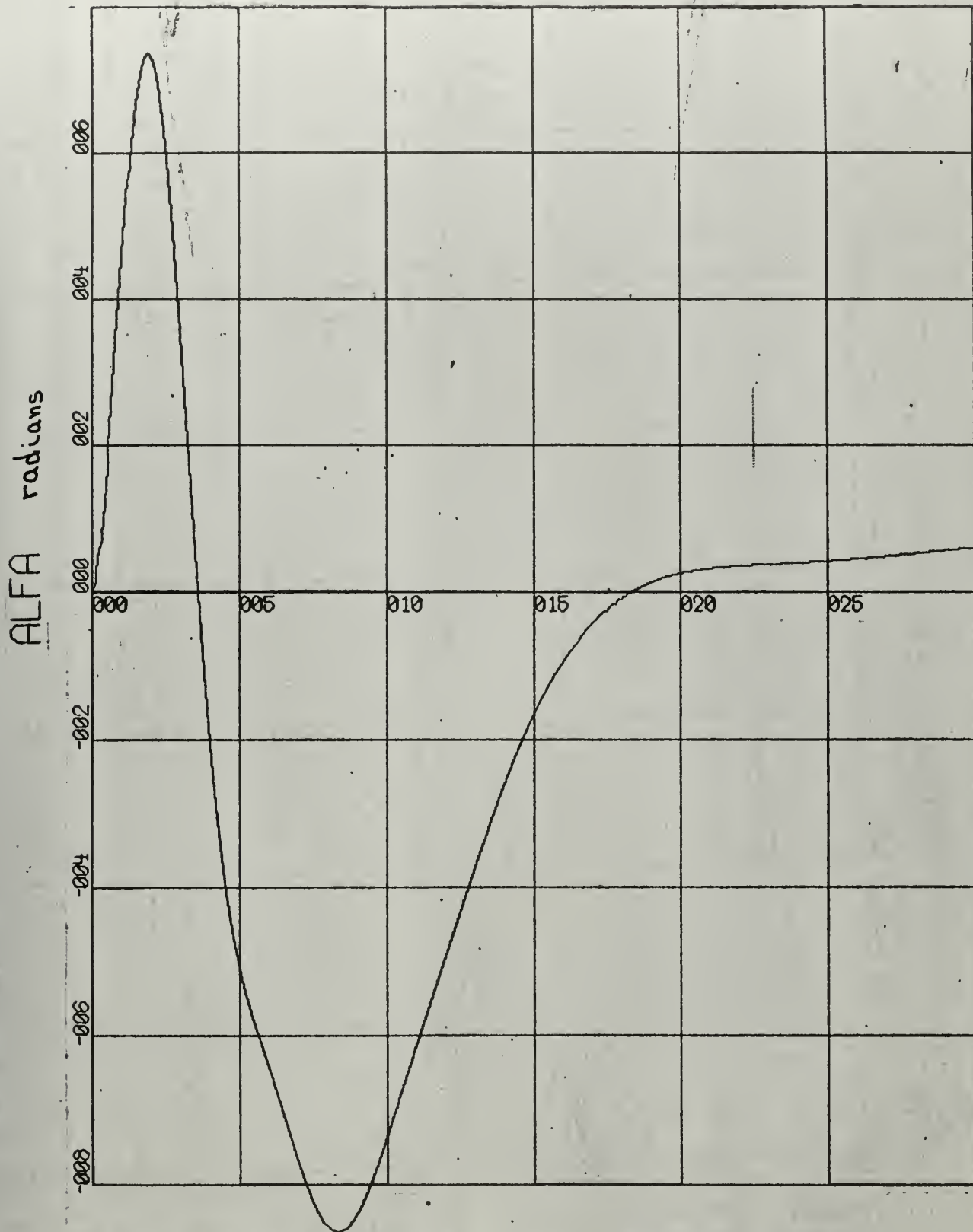
TIME sec.

(c)

Figure 15

51

X-SCALE = 2.00E+01 UNITS/INCH.
Y-SCALE = 1.00E+00 UNITS/INCH.
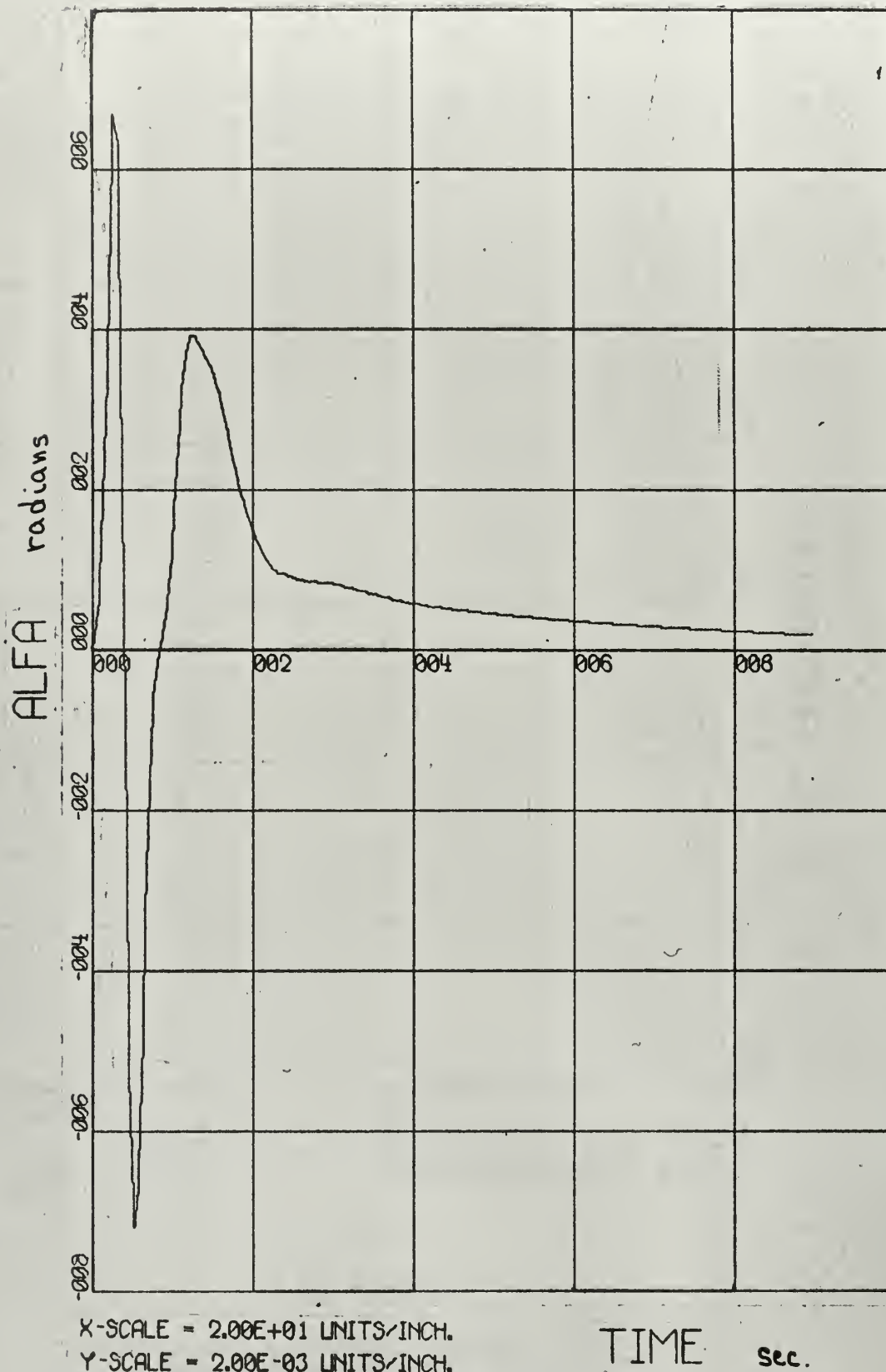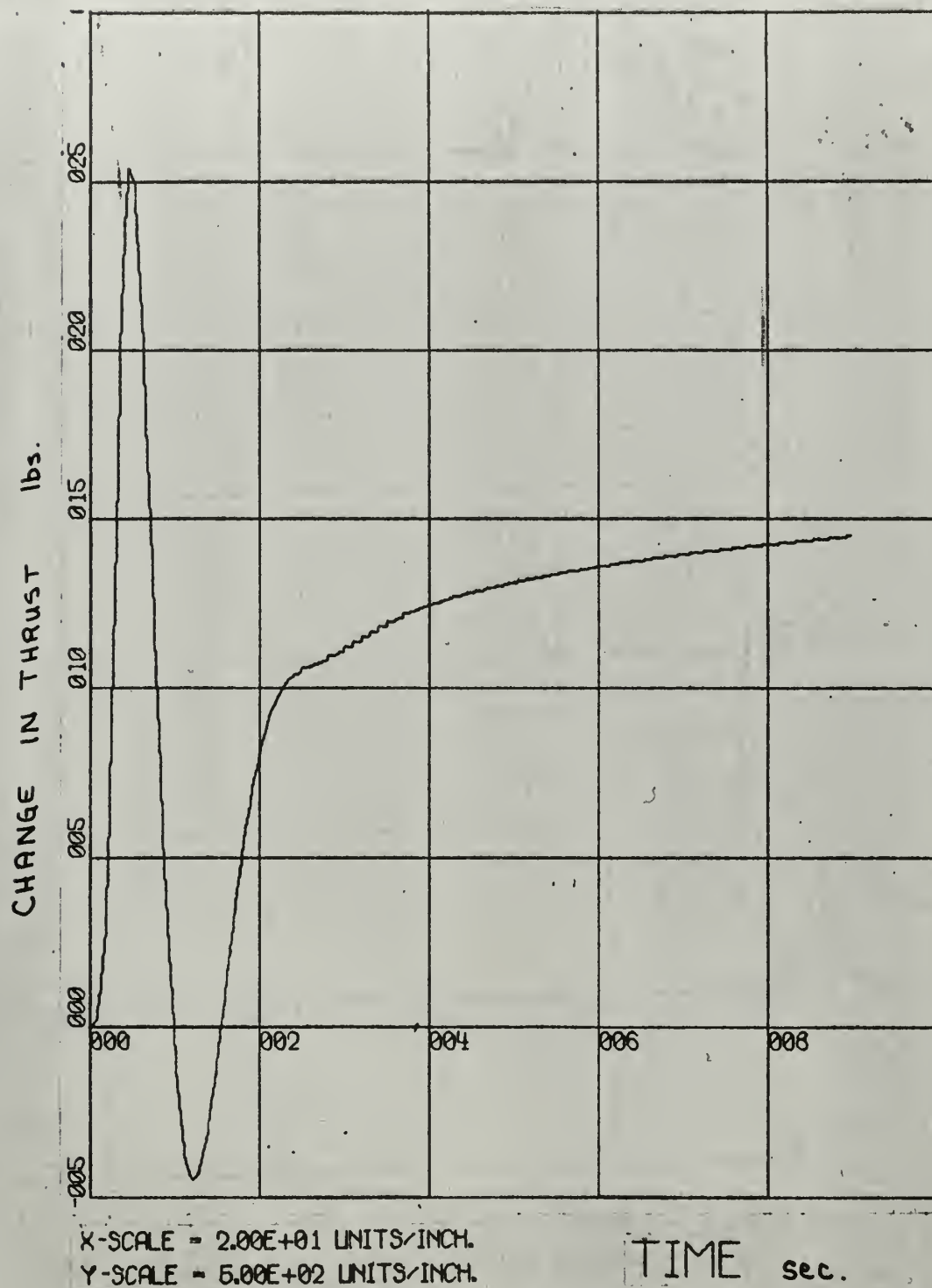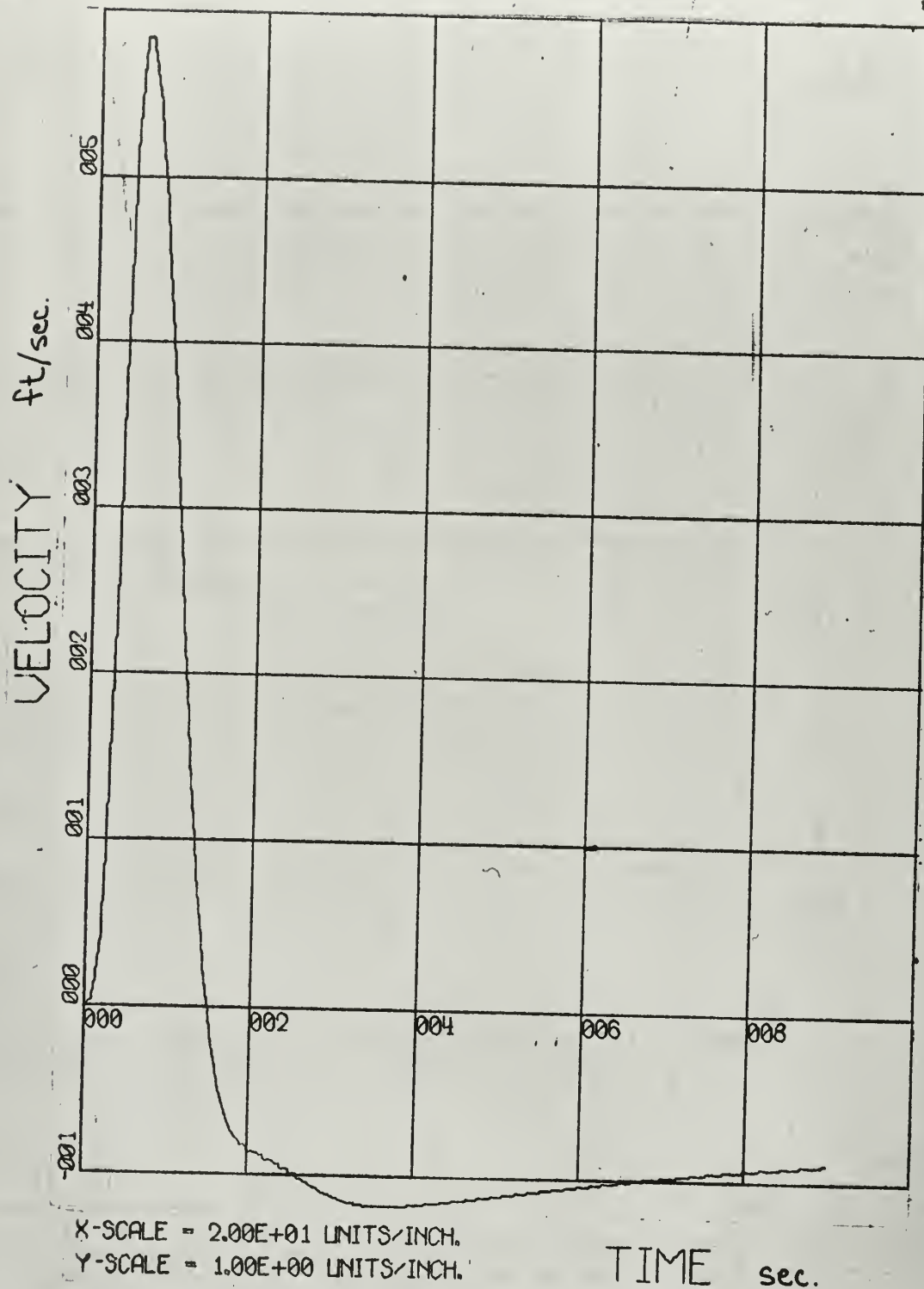
(a)

Figure 16

Aircraft-Automatic Throttle Response to
Turn with Velocity and
Angle of Attack Perturbations

52

X-SCALE = 2.00E+01 UNITS/INCH.
Y-SCALE = 5.00E-03 UNITS/INCH.

(b)

Figure 16

X-SCALE = 2.00E+01 UNITS/INCH.
Y-SCALE = 2.00E+03 UNITS/INCH.

TIME sec.

(c)

Figure 16

54
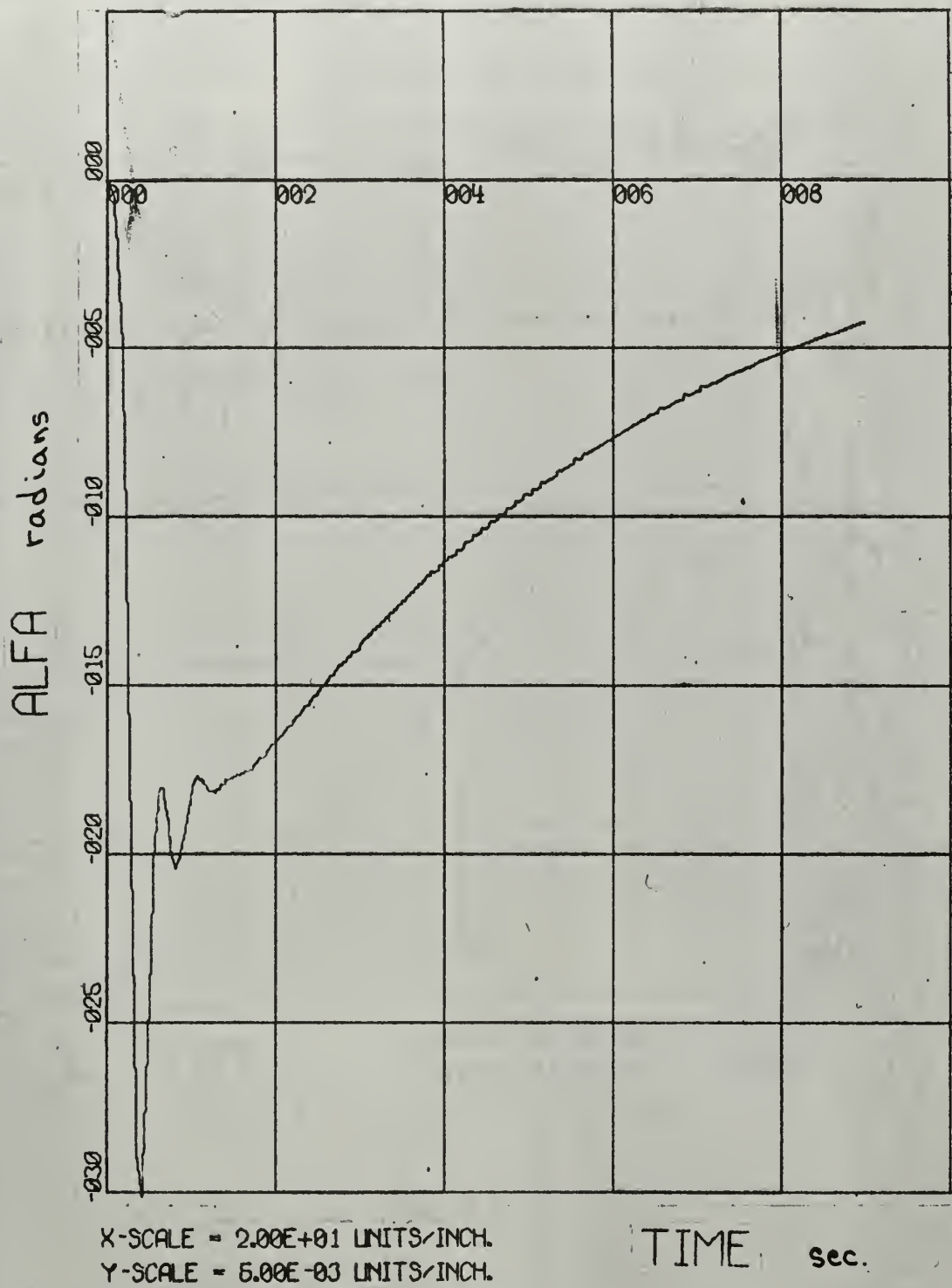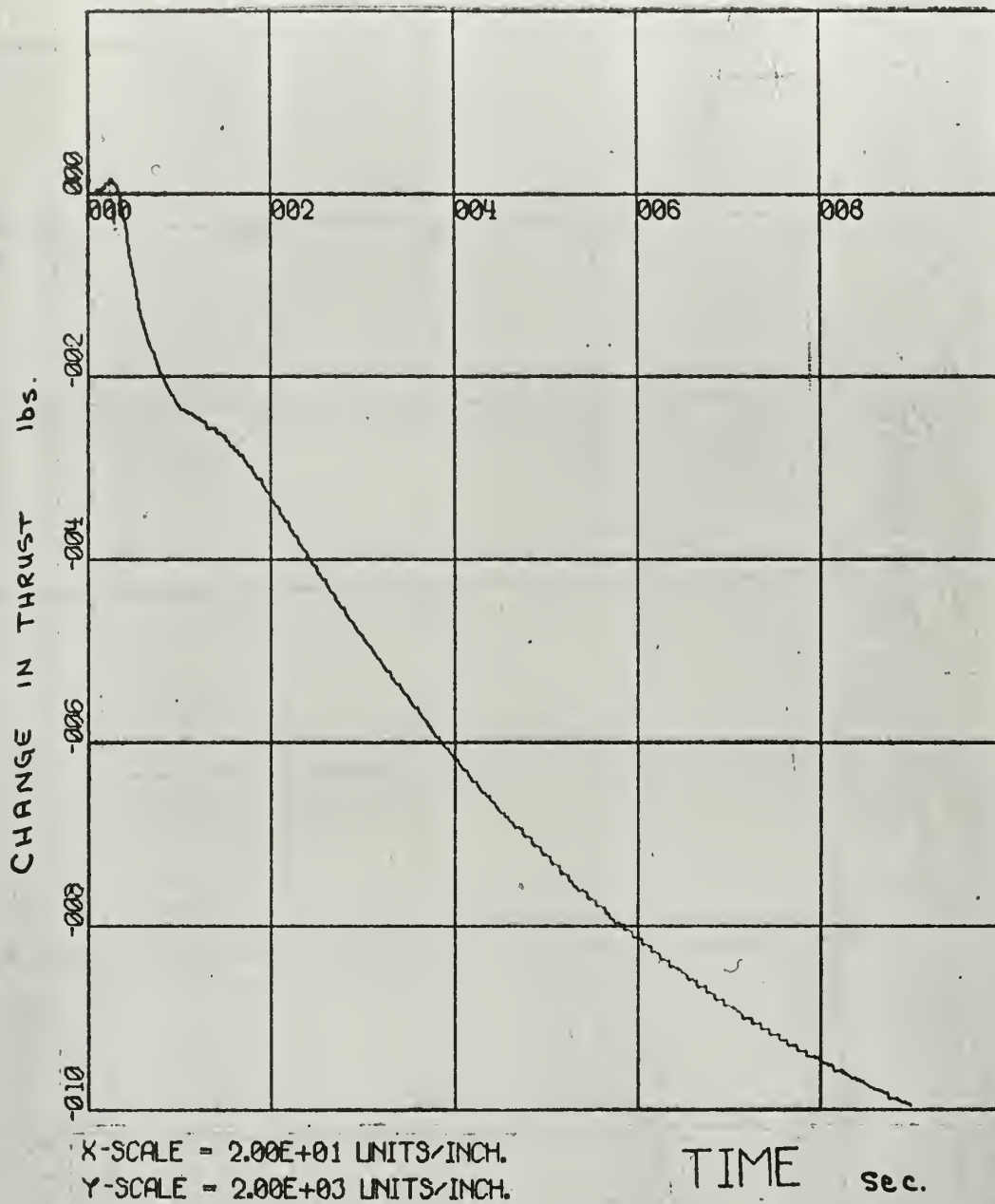
Appendix I

If the angle of attack is changed in the turn, C_L and C_D also change. The following stability derivatives vary with C_L and C_D and are developed below as functions of them.

$$\frac{X_u}{m} = -\frac{C_D \rho U_o S}{m} = -0.06 \longrightarrow C_D = .197$$

$$\frac{X_u}{m} = -.3052 \, C_D \quad 1/\text{sec}$$

$$\frac{Z_u}{m U_o} = -\frac{C_L \rho U_o S}{m U_o} = -.00113 \longrightarrow C_L = .869$$

$$\frac{Z_u}{m U_o} = -.0013 \, C_L \quad 1/\text{ft}$$

$$\frac{X_w U_o}{m} = \frac{\rho U_o^2 S}{2m} \left(1 - \frac{2 C_{L\alpha}}{\epsilon \pi A}\right) C_L$$

$$\left(1 - \frac{2 C_{L\alpha}}{\epsilon \pi A}\right) = -.106$$

$$\frac{X_w U_o}{m} = -3.8 \, C_L \quad \text{ft/sec}^2$$

$$\frac{Z_w}{m} = -\frac{\rho U_o S}{2m}\left(C_{L\alpha} + C_D\right) = -.426$$

$$C_{L\alpha} = 2.6$$

$$\frac{Z_w}{m} = -.1562 \left(2.6 + C_D\right) \quad 1/\text{sec}$$

55

Figure 2 is a plot of C_L versus C_D. Below is a derivation of a quadratic equation for C_D as a function of C_L.

$$C_D = a + b\,C_L + c\,C_L^2$$

$$.197 = a + b(.869) + c(.869)^2$$
$$.21 = a + b(.9) + c(.9)^2$$
$$.273 = a + b(1.003) + c(1.003)^2$$

$$a = \frac{\begin{vmatrix} .197 & .869 & .755 \\ .21 & .9 & .81 \\ .273 & 1.003 & 1.006 \end{vmatrix}}{\begin{vmatrix} 1.0 & .869 & .755 \\ 1.0 & .9 & .81 \\ 1.0 & 1.003 & 1.006 \end{vmatrix}} = \frac{.00041}{.00043} = .955$$

$$b = \frac{\begin{vmatrix} 1.0 & .197 & .755 \\ 1.0 & .21 & .81 \\ 1.0 & .273 & 1.006 \end{vmatrix}}{.00043} = \frac{-.00091}{.00043} = -2.12$$

$$c = \frac{\begin{vmatrix} 1.0 & .869 & .197 \\ 1.0 & .9 & .21 \\ 1.0 & 1.003 & .273 \end{vmatrix}}{.00043} = \frac{.000614}{.00043} = 1.44$$

$$C_D = .955 - 2.12\,C_L + 1.44\,C_L^2$$

Appendix III

Using the simplified relationship between elevator and angle of attack given below an equation for elevator as a function of angle can be found.

$$C_{m_\eta} \eta = C_{m_\alpha} \alpha$$

$$M_\alpha = \frac{M_{\dot{\omega}} U_o}{B} = -1.14$$

$$M_\alpha = \frac{\rho U_o^2 S \bar{c}}{2B} \quad C_{m_\alpha} = -1.14$$

$$C_{m_\alpha} = -.381$$

$$\frac{M_\eta}{B} = \frac{\rho U_o^2 S \bar{c}}{2B} \quad C_{m_\eta} = -2.5$$

$$C_{m_\eta} = -.834$$

$$\eta = \frac{C_{m_\alpha}}{C_{m_\eta}} \alpha$$

$$\eta = .46 \, \alpha$$

Below are the formulae and calculations for the thrust versus velocity at various angles of attack. The plot of these calcutions is shown in Figure 11.

$\phi = 0°$

$$D = T = \frac{1}{2} \rho V_a^2 S C_{D_a}$$

$$T = 4810 \text{ lbs.}$$

$\phi = 10°$

1. Keeping the same velocity in the turn.

$$V_t = V_a$$

$$L = \frac{1}{2} \rho V_a^2 S C_{L_a} \qquad\qquad \phi = 0°$$

$$\frac{L}{\cos \phi} = \frac{1}{2} \rho V_a^2 S C_{L_t} \qquad\qquad \text{in turn}$$

$$C_{L_t} = \frac{C_{L_a}}{\cos \phi} \qquad\qquad \text{combining}$$

$$C_{L_t} = .8824$$

from Figure 2

$$C_{D_t} = .2025$$

$$T = \frac{1}{2} \rho V_a^2 S C_{D_t}$$

$$T = 4948 \text{ lbs.}$$

2. Keeping the same angle of attack in the turn.

$$\alpha_a = \alpha_t$$

$$L = \tfrac{1}{2} \rho V_a^2 S C_{L_a} \qquad\qquad \phi = 0°$$

$$\frac{L}{\cos\phi} = \tfrac{1}{2} \rho V_t^2 S C_{L_a} \qquad\qquad \text{in turn}$$

$$V_t = \frac{V_a}{\sqrt{\cos\phi}} \qquad\qquad \text{combining}$$

$$V_t = 235.8 \ \text{ft/sec}$$

$$T = \tfrac{1}{2} \rho V_t^2 S C_{D_a}$$

$$T = 4888 \ \text{lbs.}$$

3. Flying an angle of attack corresponding to a coeffi-
    cient of lift of .875. while in the turn.

$$C_{L_t} = .875$$

from Figure 2

$$C_{D_t} = .1995$$

$$L = \tfrac{1}{2} \rho V_a^2 S C_{L_a} \qquad\qquad \phi = 0°$$

$$\frac{L}{\cos\phi} = \tfrac{1}{2} \rho V_t^2 S C_{L_t} \qquad\qquad \text{in turn}$$

$$V_t = V_a \sqrt{\frac{C_{L_a}}{C_{L_t} \cos\phi}} \qquad\qquad \text{combining}$$

$$V_t = 235 \ \text{ft/sec}$$

$$T = \tfrac{1}{2} \rho V_t^2 S C_{D_t}$$

$$T = 4917 \ \text{lbs.}$$

$\phi = 20°$

1. Keeping the same velocity in the turn.

$$V_t = V_a$$

$$C_{L_t} = \frac{C_{L_a}}{\cos \phi} = .925$$

from Figure 2

$$C_{D_t} = .223$$

$$T = \frac{1}{2} \rho V_a^2 S C_{D_t}$$

$$T = 5445 \text{ lbs}$$

2. Keeping the same angle of attack in the turn.

$$\alpha_t = \alpha_a$$

$$V_t = \frac{V_a}{\sqrt{\cos \phi}} = 241.4 \text{ ft/sec}$$

$$T = \frac{1}{2} \rho V_t^2 S C_{D_a}$$

$$T = 5123 \text{ lbs.}$$

3. Flying an angle of attack corresponding to a coefficient of lift of .8824 while in the turn.

$$C_{L_t} = .8824$$

from Figure 2

$$C_{D_t} = .2025$$

$$V_t = V_a \sqrt{\frac{C_{L_a}}{C_{L_t} \cos \phi}} = 239.55 \text{ ft/sec}$$

$$T = \frac{1}{2} \rho V_t^2 S C_{D_t}$$

$$T = 5183 \text{ lbs.}$$

$\phi = 30°$

1. Keeping the same velocity in the turn.

   $V_t = V_a$

   $C_{L_t} = \dfrac{C_{L_a}}{\cos\phi} = 1.003$

   from Figure 2

   $C_{D_t} = .273$

   $T = \dfrac{1}{2} \rho V_a^2 S C_{D_t}$

   $T = 6666 \text{ lbs.}$

2. Keeping the same angle of attack in the turn.

   $\alpha_t = \alpha_a$

   $V_t = \dfrac{V_a}{\sqrt{\cos\phi}} = 251.5 \text{ ft/sec}$

   $T = \dfrac{1}{2} \rho V_t^2 S C_{D_a}$

   $T = 5560 \text{ lbs.}$

3. Flying an angle of attack corresponding to a coefficient of lift of .8824 while in the turn.

   $C_{L_t} = .8824$

   from Figure 2

   $C_{D_t} = .2025$

   $V_t = V_a \sqrt{\dfrac{C_{L_a}}{C_{L_t} \cos\phi}} = 249.5 \text{ ft/sec}$

   $T = \dfrac{1}{2} \rho V_t^2 S C_{D_t}$

   $T = 5625 \text{ lbs.}$

Appendix V

This appendix contains calculations of the possible perturbations introduced to the velocity and the angle of attack while.

$$L = W = \frac{1}{2} \rho V^2 S C_L$$

$$V^2 = \frac{2W}{\rho S C_L}$$

$$2V \frac{dV}{dC_L} = -\frac{2W}{\rho S C_L^2}$$

$$dV = \Delta V = u \qquad dC_L = \Delta C_L$$

$$u = -\frac{W \; \Delta C_L}{\rho V S C_L^2}$$

$$\Delta C_L = \frac{C_L}{\cos \phi} - C_L$$

$$u = -\frac{W}{\rho V S C_L} \left(\frac{1}{\cos \phi} - 1 \right)$$

$$u = -121.4 \left(\frac{1}{\cos \phi} - 1 \right)$$

$$\Delta C_L = C_{L\alpha} \Delta \alpha$$

$$\Delta \alpha = \frac{\Delta C_L}{C_{L\alpha}}$$

$$\Delta \alpha = \alpha$$

$$\alpha = \frac{C_L}{C_{L\alpha}} \left(\frac{1}{\cos \phi} - 1 \right)$$

$$\alpha = .334 \left(\frac{1}{\cos \phi} - 1 \right)$$

Appendix VI

Utilizing the figure below, the equations following are easily derived.
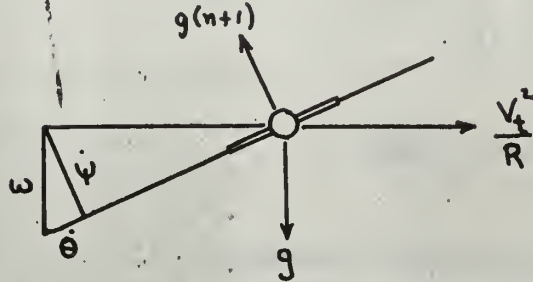
X-axis	$T = D$
Y-axis	$\dfrac{V_T^2}{R} \cos \phi = g \sin \phi$
Z-axis	$g(n+1) = g \cos \phi + \dfrac{V_t^2}{R} \sin \phi$
Horizontal plane	$g \sin \phi (n+1) = \dfrac{V_T^2}{R} = \omega V_T$
Turn rates	$\dfrac{\dot{\Theta}}{\omega} = \sin \phi$
	$\cos \phi = \dfrac{1}{n+1}$

Combining the above equations the following pitch rate perturbation can be obtained.

$$\dot{\Theta} = \frac{g}{V_t} \, n \, \frac{(n+2)}{(n+1)}$$

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This thesis is a continuation of a series of studies on automatic throttle compensation systems begun in 1963, by Lieutenant Commander G. R. Bell. Previous investigations have been concerned with flight in a wings level attitude. This study will consider the effects of turning flight on the system. The system under consideration here is the Specialties, Incorporated; Automatic Power Compensator, (APC), an automatic throttle in use in the F-8 and F-4 aircraft. It utilizes feedbacks of angle of attack and normal acceleration. It was found important to differentiate between spurious perturbations and those perturbations from steady state introduced due to the aircraft entering a turn.

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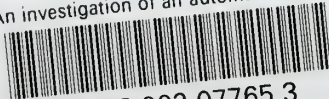
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